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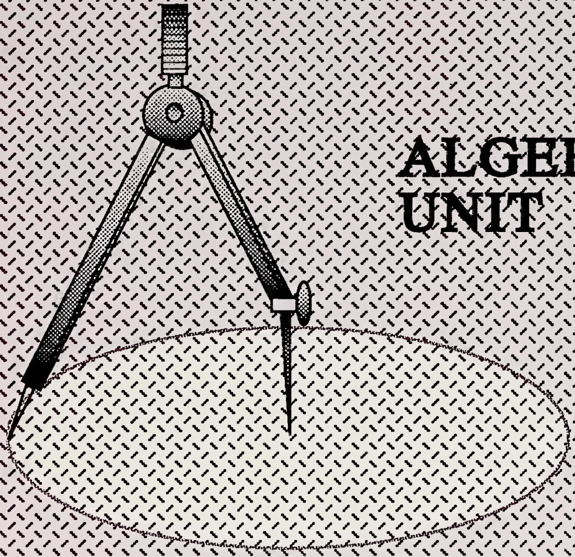
MATHEMATICS 23



ALGEBRA
UNIT 2

Distance
Learning

Alberta
EDUCATION

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Welcome

Distance
Learning

You have chosen an alternate form of learning that allows you to work at your own pace. You will be responsible for your own schedule, for disciplining yourself to study the units thoroughly, and for completing your units regularly. We wish you much success and enjoyment in your studies.

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Mathematics 23 Student Module Unit 2 Algebra Alberta Distance Learning Centre ISBN No. 0-7741-0787-1 *1992

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General Information

This information explains the basic layout of each booklet.

- **What You Already Know** and **Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.

- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.

- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.

- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.

- **Extensions** gives you the opportunity to take the topic one step further.

- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.

- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics **(Appendix B, etc.)**.

Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

An explanation of what they mean is written beside each visual cue.

Key Idea
- flagging important ideas

Audiotape
- learning by listening to an audiotape

What You Already Know
- reviewing what you already know

Another View
- exploring different perspectives

Computer Software
- learning by using computer software

Review
- studying previous concepts

Solutions
- correcting the activities

Videotape
- learning by viewing a videotape

Introduction
- introducing the unit

Extra Help
- providing additional study

Print Pathway
- choosing a print alternative

What Lies Ahead
- previewing the unit

Extensions
- going on with the topic

Calculator
- using your calculator

Exploring the Topic
- actively learning new concepts

What You Have Learned
- summarizing what you have learned

Mathematics 23

Course Overview

Mathematics 23 contains 8 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Powers and Radicals	10%
Unit 2 Algebra	**12%**
Unit 3 Mathematics of Finance	4%
Unit 4 Linear Relations	12%
Unit 5 Systems of Equations	16%
Unit 6 Geometry	16%
Unit 7 Trigonometry	16%
Unit 8 Statistics	14%
	100%

Unit Assessment

After completing the unit you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal your teacher will determine what this assessment will be. It may be

Unit assignment - 50%
Supervised unit test - 50%

Introduction to Algebra

This unit covers topics dealing with Algebra. Each topic contains explanations, examples, and practice to assist you in understanding algebra. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help.** If you would like to extend your knowledge of the topic, there is a section called **Extensions.**

You can evaluate your understanding of each topic by working through the activities. Answers are found in the Solutions in the **Appendix.** In several cases there is more than one way to do the question.

Unit 2 Algebra

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- What Lies Ahead
- Exploring Topic 1
- Extra Help
- Extensions

25% **Topic 2:** Solving Linear Inequalities and Graphing Their Solutions 24
- Introduction
- What Lies Ahead
- Exploring Topic 2
- Extra Help
- Extensions

25% **Topic 3:** Factoring Polynomials of the Form $ax^2+bx+c;\ a,b,c \in I$ 35
- Introduction
- What Lies Ahead
- Exploring Topic 3
- Extra Help
- Extensions

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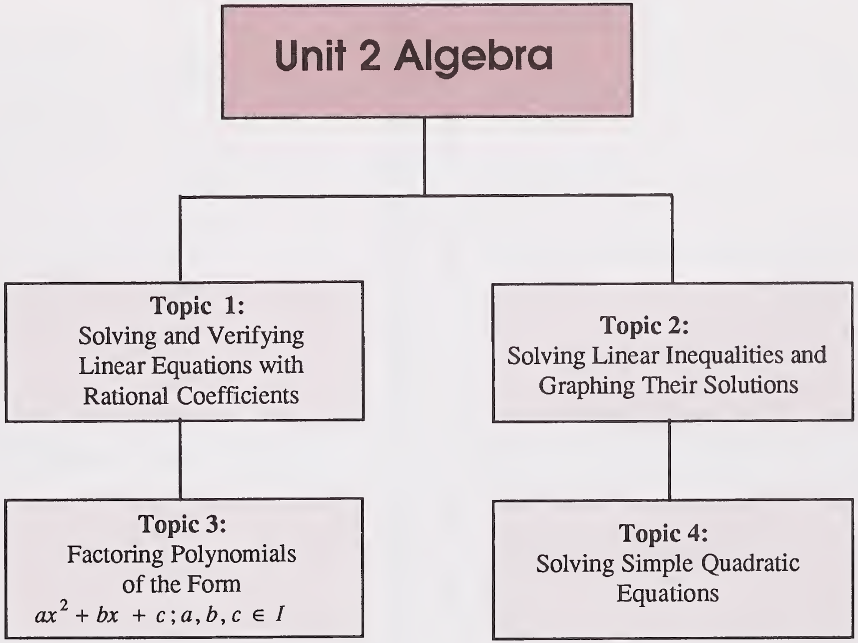
Algebra

A carpenter, with the proper raw materials and tools, can build a simple house or towering office building. In a similar way, mathematicians using polynomials and their operations as tools can build algebraic expressions or equations. These expressions may be simple or they may be complex, but the basic skills necessary to work with polynomials are the same.

Algebra skills are used throughout mathematics. In this unit, you will review your skills with polynomials, factoring, and solving equations, and develop new skills that will be necessary as you study more complex algebra.



Unit 2 Algebra

Topic 1:
Solving and Verifying Linear Equations with Rational Coefficients

Topic 2:
Solving Linear Inequalities and Graphing Their Solutions

Topic 3:
Factoring Polynomials of the Form
$ax^2 + bx + c ; a, b, c \in I$

Topic 4:
Solving Simple Quadratic Equations

What You Already Know

Refresh your memory!

Do you remember the following facts?

1. To multiply fractions, reduce first, if possible. For example,

$$\frac{4}{\cancel{5}_{1}} \times \overset{5}{\cancel{25}} = \frac{4}{1} \times \frac{5}{1}$$
$$= 20.$$

2. To find the **lowest common denominator** (L.C.D.) for two or more fractions, determine the smallest number that all denominators divide into. For example,

$$\frac{1}{3}, \frac{1}{9}, \frac{5}{6}.$$

List the factors for each denominator. The L.C.D. is the smallest number they have in common.
3: 3, 6, 9, 12, 15, 18, 24, 27, . . .
6: 6, 12, 18, 24, 30, . . .
9: 9, 18, 27, 36, . . .
L.C.D. is 18

3. A number can be written as a **prime factorization.** For example,

$$24 = \underbrace{2 \times 2 \times 2 \times 3}_{\text{all prime factors}}.$$

4. The **greatest common factor** (G.C.F.) for a set of numbers is the largest number that divides into **each** of the numbers. For example,

$$18 = 2 \times 3 \times 3$$
$$36 = 2 \times 2 \times 3 \times 3$$
$$72 = 2 \times 2 \times 2 \times 3 \times 3.$$

Take the least number of each factor that is in all of the numbers.

$$\text{G.C.F.} = 2 \times 3 \times 3$$
$$= 18$$

5. The Product of Powers Property.

For example, $a^m \times a^n = a^{m+n}$

$$3^3 \times 3^4 = 3^{3+4}$$
$$= 3^7.$$

The Quotient of Powers Property.

For example, $a^m \div a^n = a^{m-n}$

$$\frac{3^7}{3^4} = 3^{7-4}$$
$$= 3^3.$$

A prime number is a number that can be divided evenly only by one and the number itself.
Examples are: 2, 3, 5, 7, 11, 13, etc.

6. To solve an equation, use inverse or opposite operations. For example,

$$x-3=9$$
$$x-3+3=9+3 \qquad \text{add 3 to both sides}$$
$$x=12.$$

7. One can evaluate an expression containing variables if specific values of the variables are given.

For example, if $x=2$ and $y=3$, then

$$\begin{aligned}2x+5y&=2\times2+5\times3\\&=4+15\\&=19.\end{aligned}$$

If $p(x)=3x^2+2x-1$, then

$$\begin{aligned}p(2)&=3\times2^2+2\times2-1\\&=3\times4+4-1\\&=12+3\\&=15.\end{aligned}$$

8. To add polynomials, add like terms.
For example,

$$\begin{aligned}&\left(2x^2+5xy+y^2\right)+\left(x^2+xy+y^2\right)\\&=\left(2x^2+x^2\right)+(5xy+xy)+\left(y^2+y^2\right)\\&=3x^2+6xy+2y^2.\end{aligned}$$

(If a term does not have a numerical coefficient it is understood to be 1.)

9. To subtract polynomials, subtract like terms.
For example,

$$\begin{aligned}&\left(2x^2+5xy+y^2\right)-\left(x^2+xy+y^2\right)\\&=2x^2+5xy+y^2-x^2-xy-y^2\\&=\left(2x^2-x^2\right)+(5xy-xy)+\left(y^2-y^2\right)\\&=x^2+4xy.\end{aligned}$$

10. To multiply two or more monomials, multiply the numerical coefficient by numerical coefficient and variable by variable. For example,

$$\begin{aligned}&\left(3xy^2\right)\left(5x^2y^4z\right)\\&=(3\times5)\left(x\times x^2\right)\left(y^2\times y^4\right)(z)\\&=15x^3y^6z.\end{aligned}$$

11. To multiply a monomial by a polynomial, apply the distributive property and multiply every term of the polynomial by the monomial. For example,

$$\begin{aligned}&5a\left(3a^2+4ab+b^2\right)\\&=(5a)\left(3a^2\right)+(5a)(4ab)+(5a)\left(b^2\right)\\&=15a^3+20a^2b+5ab^2.\end{aligned}$$

12. To multiply two binomials, apply the distributive property twice. Remember FOIL? For example,

$$\begin{aligned}(x+1)(x-2)&=(x\times x)+(x\times-2)+(1\times x)+(1\times-2)\\&=x^2-2x+x-2\\&=x^2-x-2.\end{aligned}$$

13. To multiply two polynomials, apply the distributive property. Multiply each term in the second polynomial by each term in the first polynomial. For example,

$$(3x+1)(x^2-2x+3)$$

$$=(3x\times x^2)+(3x\times -2x)+(3x\times 3)+(1\times x^2)+(1\times -2x)+(1\times 3)$$

$$=3x^3-6x^2+9x+x^2-2x+3$$

$$=3x^3-5x^2+7x+3.$$

14. To factor a trinomial, remove any common factor and factor the trinomial into two binomials.
For example, $3x^2+3x-6$

$$=3(x^2+x-2)$$

$$=3(x-1)(x+2).$$

15. Use the 4-step problem-solving approach.

Step 1: Understand the problem. Define the unknown using a variable.

Step 2: Develop a plan. Find an equation to solve for the unknown.

Step 3: Carry out the plan. Solve the equation.

Step 4: Look back. Does the answer fit the situation? Check your answer.

Here is an example: Find two consecutive numbers such that the sum of two times the smaller and three times the larger is 113.

Step 1: Understand the problem.
Consecutive means the two numbers differ by one.
Let the numbers be x and $x + 1$.
Two times the smaller number is $2x$.
Three times the larger number is $3(x + 1)$.

Step 2: Develop a plan.
Since the sum is 113,
$2x+3(x+1)=113$.

Step 3: Carry out the plan.

$$\begin{aligned}2x+3(x+1)&=113\\2x+3x+3&=113\\5x&=110\\x&=22\\\therefore x+1&=23.\end{aligned}$$

Step 4: Look back. Does the answer fit the situation? Check the answer.

Are the numbers consecutive? Yes.
Does twice the smaller number plus three times the larger number equal 113?
$2(22)+3(23)=113$ ✓ check

The numbers are 22 and 23.

Now that you have looked at material that you have studied previously, turn to the **Review** on the next page to confirm your understanding of this material.

Review

Try the following review questions.

1. Multiply the following.

   a. $\frac{3}{5} \times 75$ b. $\frac{11}{9} \times 45$

2. Find the lowest common denominator for the fractions below.

   $\frac{5}{6}, \frac{7}{10}, \frac{3}{14}$

3. Express each number as a product of **prime** factors.

   a. 46 b. 63

4. Find the greatest common factor for each pair of terms.

   a. 36, 54 b. $9x^2y^3, 27xy^2$

5. Simplify the following:

   a. $7^2 \times 7^5$

   b. $x \times x^3 \times x^7$

   c. $\frac{-28x^4y^3}{7x^2y}$

6. Solve for x.
   a. $4x = 16$
   b. $x + 11 = 37$
   c. $3x - 2 = 14$

7. Evaluate $5x - 3yx + y^2$ for $x = 2, y = 1$.

8. If $P(x) = 5x^2 - 3x + 7$, then find $P(3)$.

9. Simplify $(2x^2 - 3x + 7) + (x^2 - 4x - 8)$.

10. Simplify $(2x^2 - 5x + 8) - (x^2 - 3x + 2)$.

11. Multiply $(4x^2y)(21xy^3)$.

12. Multiply $3(2x^2 - 5y) + 2x(3x + y)$.

13. Multiply $(2x + 1)(x - 3)$.

14. Multiply $(2x - 3)(x^2 - x + 2)$.

15. Factor $2x^2 + 4x - 30$.

16. Three times the length of a table plus 9 cm is 36 cm. Find the length of the table.

Now go to the **Review** solutions in the **Appendix**.

Topic 1 Solving and Verifying Linear Equations

Introduction

The Force (F) in newtons of an object is 9.8 times its mass in kg. The equation $F = 9.8 \times m$ (where m = mass) describes the relationship between mass and weight. If you know the mass of an object, you can find its weight. Equations are tools which you can use to answer questions, to solve problems.

What Lies Ahead

Throughout the topic you will learn to

1. translate English sentences into algebra
2. solve and verify simple linear equations with integral coefficients
3. solve and verify simple linear equations with rational coefficients

Now that you know what to expect, turn the page to begin your study of solving linear equations which have rational coefficients.

Exploring Topic 1

Activity 1

Translate English sentences into algebra.

Mathematics is a universal language. Universal symbols are used to represent things such as variables, unknowns, and operations. For example, + is used to represent addition and x may be used to represent a certain number. It is important that you can translate English sentences into the language of algebra. If the facts of a problem can be translated into algebraic symbols, then you can solve your problem algebraically.

Now look at some operation symbols.

1. The + symbol is equivalent to such phrases as
   "the sum of"
   "added to"
   "increased by"
   "more than"

English phrases or sentences	Mathematical phrase
the sum of x and 5	$x+5$
3 is added to y	$y+3$
4 more than x	$x+4$
7 increased by a	$7+a$

2. The – sign is equivalent to such phrases as
   "the difference of . . . from . . ."
   "less than"
   "decreased by"

English phrases or sentences	Mathematical phrase
2 is subtracted from x	$x-2$
the differencee of x from y	$y-x$
5 is decreased by x	$5-x$
9 less than a	$a-9$

3. The × sign is equivalent to such phrases as
   "the product of"
   "times"
   "multiplied"

English phrases or sentences	Mathematical phrase
4 times x	$4\times x$
y is multiplied by 5	$5\times y$
the product of a and b	$a\times b$
x is doubled	$2\times x$

4. The ÷ sign is equivalent to such phrases as
   "divided by"
   "the quotient of"
   "over"

English phrases or sentences	Mathematical phrase
a divided by b	$\frac{a}{b}$ or $a\div b$
the quotient of x by 3	$\frac{x}{3}$ or $x\div 3$
2 over 3	$\frac{2}{3}$ or $2\div 3$

A mathematical phrase can be built step by step. In some cases, you are called upon to build a mathematical phrase, but the instructions are not spelled out quite so clearly and precisely. You must sort out the instructions yourself and decide in which order the operations are to be performed on the unknown. For example, if you translate 8 more than the quotient of a certain number by 5 into a mathematical phrase, the first step is let $x =$ a certain number. Then the quotient of a certain number by 5 is $\frac{x}{5}$. Now 8 is added to the phrase to complete the translation. The mathematical phrase you want is $\frac{x}{5} + 8$.

Do you want to try some of these questions on your own?

Do the following exercises.

1. Translate each of the following into a mathematical phrase.

   a. 3 more than a certain number
   b. 7 less than a certain number
   c. 8 is divided by a certain number
   d. 6 times a certain number
   e. the difference of 5 and a certain number

2. There are two quantities in each of the following situations. Choose a variable to represent one and express the others in terms of the first. Do at least two of the following problems.

   a. Jim is two times my age.
   b. Jean earns $300 a month more than Joe.
   c. The length of rectangle is 5 cm longer than the width.
   d. The number of boys in Ms. Smith's class is 4 less than twice the number of girls.

For solutions to **Activity 1**, turn to the **Appendix, Topic 1**.

Activity 2

Solve and verify simple linear equations with integral coefficients.

You have learned how to translate an English phrase into a mathematical phrase. Now move one step further. Suppose you have a practical problem. In order to solve your problem, you must be able to translate the problem situation into mathematical phrases.

You always begin by letting some letter represent the unknown quantity of the problem. Then you set up the equation and solve it by applying the skills you have acquired. The following examples will show you the procedure.

Example 1

John is a dishwasher salesman. His monthly salary is $1000 plus $55 commission per dishwasher sold. His last paycheque was $2925. How many dishwashers did he sell?

Solution:

Follow the 4-step procedure.

Step 1: Understand the problem.

Let x = number of dishwashers sold.

Commission = $55$x$

Step 2: Develop a plan.
Total salary = $1000 + $55$x$ = $2925

Step 3: Carry out the plan.
Solve the equation.

$$1000 + 55x - 1000 = 2925 - 1000 \quad \text{(Subtract 1000 from both sides.)}$$

$$55x = 1925$$

$$\frac{55x}{55} = \frac{1925}{55} \quad \text{(Divide both sides by 55.)}$$

$$x = 35$$

Step 4: Look back. Does the answer fit the situation? Check the answer.
Substitute x = (35) into the original equation.

LS	RS
$1000 + 55(35)$	2925
$1000 + 1925$	
2925	

LS = RS

∴ John sold 35 dishwashers last month.

Every time you solve a word problem, you have to solve an equation. When solving an equation, you may have to apply the properties like the distributive property, the associative property, and the basic operations. Look at an example.

Example 2

Solve $3(x-2)+x=5(x-1)+3$.

Solution:

$3x-6+x=5x-5+3$	Use the distributive property.
$4x-6=5x-2$	Simplify.
$4x-6-5x=5x-2-5x$	Subtract $5x$ from both sides.
$-x-6=-2$	
$-x-6+6=-2+6$	Add 6 to both sides.
$-x=4$	
$x=-4$	Divide both sides by (–1).

The next example involves linear equations with parentheses.

Example 3

Peter is a part-time worker. For the last three weeks he earned \$1373. He earned \$25 more the first week than the second week. In the third week, he earned \$2 less than the second week.
How much did he earn each week?

Solution:

Step 1: Understand the problem.
Let x = amount earned in the second week.
Then $x + 25$ = amount earned in the first week.
$x - 2$ = amount earned in the third week.

Step 2: Develop a plan.
$(x+25)+x+(x-2)=\$1373$

Step 3: Carry out the plan.

$x+25+x+x-2=1373$	
$3x+23=1373$	(Simplify.)
$3x=1373-23$	(Subtract 23 from both sides.)
$3x=1350$	
$x=\dfrac{1350}{3}$	(Divide both sides by 3.)
$=450$	

$$x+25=450+25$$
$$=475$$
$$x-2=450-2$$
$$=448$$

Step 4: Look back. Does the answer fit the situation?
Check your answer.
Substitute 450 into the original equation.

LS	RS
$(x+25)+x+(x-2)$	1373
$450+25+450+(450-2)$	
$475+450+450-2$	
1373	

LS = RS

Peter earned \$ 475, \$ 450, and \$ 448 in the three weeks.

There are many different types of problems. It is impossible to show you all of them. The following are just two more examples.

Example 4

Find two consecutive even numbers such that the difference of 4 times the smaller by 2 times the larger is 16.
Solution:

Step 1: Understand the problem.
Let x = smaller number.
Then $(x + 2)$ = larger number.

Step 2: Develop a plan.
$4(x)-2(x+2)=16$

Step 3: Carry out the plan.

$$\begin{aligned} 4x-2x-4&=16 && \text{(Use the distributive property.)}\\ 2x-4&=16 && \text{(Simplify.)}\\ 2x&=20 && \text{(Add 4 to both sides.)}\\ x&=\frac{20}{2} && \text{(Divide both sides by 2.)}\\ x&=10\\ \therefore x+2&=10+2\\ &=12 \end{aligned}$$

Step 4: Look back.

LS	RS
$4x-2(x+2)$	16
$4(10)-2(10+2)$	
$40-20-4$	
$20-4$	
16	

LS = RS
The two numbers are 10 and 12.

Example 5

Lisa is 10 years younger than Kana. Two years from now, Kana will be twice as old as Lisa. How old is Kana now?

Solution:

Step 1: Understand the problem.
Let x = Lisa's age now.
$x + 10$ = Kana's age now.
Two years from now:
Lisa's age = $x + 2$
Kana's age = $(x + 10) + 2$
$= x + 12$

Step 2: Develop a plan.
$(x + 12) = 2 (x + 2)$ (Two years from now,
Kana's age = 2 × Lisa's age.)

Step 3: Carry out the plan.

$$\begin{aligned} x+12&=2x+4\\ x+12-x&=2x+4-x && \text{(Subtract } x \text{ from both sides.)}\\ 12&=x+4\\ 12-4&=x+4-4 && \text{(Subtract 4 from both sides.)}\\ 8&=x\\ x&=8\\ x+10&=18 \end{aligned}$$

Step 4: Look back. Does the answer fit the situation? Check the answer.

LS	RS
$x+12$	$2(x+2)$
$8+12$	$2(8+2)$
20	$2(10)$
	20

$LS = RS$

Therefore, Kana is 18 years old.

Now, would you like to try a couple of problems? Do the following:

1. Solve and verify either a or b.
   a. $3x-2(x+4)=5-3(x-1)$
   b. $2+3(x-5)=x-2(x+3)$

Do any four of the following five questions.

2. The larger of two numbers is 8 more than 2 times the smaller number. If their difference is 15, find each number.

3. In ΔABC, The measure of $\angle A$ is $5°$ more than the measure of $\angle B$. The measure of $\angle C$ is $35°$ less than the measure of $\angle B$. What is the measure of each angle? (Remember that the sum of the measures of the three angles of any triangle is always $180°$.)

4. A mother is 8 years more then 3 times her son's age. Four years ago, she was 11 times as old as her son. How old is the mother?

5. The length of a rectangle is 5 cm longer than the width. If the perimeter of the rectangle is 90 cm, find the dimensions of the rectangle.

6. In a bag of coins there are 5 more dimes than nickels and 2 less quarters than nickels. If the coins are worth a total of $2.00, find the number of each kind of coin.

✓ For solutions to **Activity 2**, turn to the **Appendix, Topic 1**

Activity 3

Solve and verify simple linear equations with rational coefficients.

How many $\frac{1}{3}$ kg boxes can a truck carry if the capacity of the truck is 1200 kg? In order to solve this problem, you have to build a linear equation and the coefficient of this equation will be a rational number. The process of solving an equation is used over and over again in mathematics. The equation may be simple or it may be complex, but the process used in solving is always the same. Solving an equation means finding the value of the variable that makes the question true. To solve an equation, you must undo what has been done to build it. Now study the question asked above. Use the 4-step problem-solving approach.

Example 6

How many $\frac{1}{3}$ kg boxes can a truck carry if the capacity of the truck is 1200 kg?
Solution:

Step 1: Let x be the number of boxes. The total weight of all the boxes is $\frac{1}{3}x$ kg.

Step 2: Since the capacity of the truck is 1200 kg.

$$\frac{1}{3}x = 1200$$

Step 3: Solve the equation.
Since x is multiplied by 1 and divided by 3, you must multiply both sides by 3.
(Undo what has been done!)

$$(3)\left(\frac{1}{3}x\right) = (3)(1200)$$

$$x = 3600$$

Step 4: Check the solution.
If $x = 3600$

LS	RS
$\frac{1}{3}x$	1200
$\frac{1}{3}(3600)$	
1200	

LS = RS

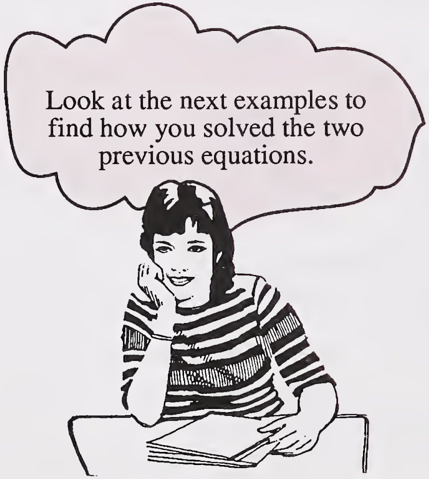
The truck can carry 3600 boxes.

Example 6 is a simple one. The equation has only one term with one variable and only one rational coefficient. If the equation has more than one term which contains a variable, what method is used to solve it? For example,

$$\frac{x}{3} - \frac{x}{5} = 2 \quad \text{or} \quad \frac{x-2}{3} + \frac{x+1}{4} = \frac{1}{5}$$

are more complicated equations.

To solve this kind of equation, the first thing you do is to eliminate the fractions by multiplying each term on each side by the lowest common denominator of all the fractions.


Look at the next examples to find how you solved the two previous equations.

4-step problem-solving approach:
Step 1: Understand the problem.
Step 2: Develop a plan.
Step 3: Carry out the plan.
Step 4: Look back. Check the answer.

Example 7

Solve $\frac{x}{3}-\frac{x}{5}=2$.

Solution:

The common denominator of 3 and 5 is 15. Multiply each term on each side of the equation by 15.

$$\overset{5}{\cancel{15}}\times\frac{x}{\underset{1}{\cancel{3}}}-\overset{3}{\cancel{15}}\times\frac{x}{\underset{1}{\cancel{5}}}=15\times 2$$

$$5x-3x=30$$

$$2x=30$$

$$x=\frac{30}{2}$$

$$x=15$$

Sometimes it is not necessary to eliminate fractions to solve a linear equation with rational coefficients. For example,

$$\frac{1}{2}x-\frac{x}{3}=\frac{2}{5}$$

Since the lowest common denominator of 2 and 3 is 6, the equation can be changed to

$$\frac{3x}{6}-\frac{2x}{6}=\frac{2}{5}$$

$$\frac{x}{6}=\frac{2}{5}$$

$$x=\frac{2\times 6}{5}$$

$$x=\frac{12}{5}$$

This is just another method. If you don't like it, you don't have to use it. Take a look at another example.

Example 8

Solve $\frac{x-2}{3}+\frac{x+1}{4}=\frac{1}{5}$.

Solution:

The lowest common denominator of 3, 4, and 5 is 60.
Multiply every term on each side by 60.

$$\overset{20}{\cancel{60}}\times\frac{x-2}{\underset{1}{\cancel{3}}}+\overset{15}{\cancel{60}}\times\frac{x+1}{\underset{1}{\cancel{4}}}=\overset{12}{\cancel{60}}\times\frac{1}{\cancel{5}}$$ (Reduce to remove fractions.)

$$20(x-2)+15(x+1)=12\times 1$$

$$20x-40+15x+15=12$$ (Apply distributive property.)

$$35x-25=12$$ (Simplify.)

$$35x=37$$ (Add 25 to both sides.)

$$x=\frac{37}{35}$$ (Divide both sides by 35.)

Important Tip:
Put **brackets** around numerators that contain more than one term.

Verify $x = \frac{37}{35}$.

LS	RS
$\frac{x-2}{3} + \frac{x+1}{4}$	$\frac{1}{5}$
$\frac{\frac{37}{35}-2}{3} + \frac{\frac{37}{35}+1}{4}$	
$\frac{\frac{37}{35}-\frac{70}{35}}{3} + \frac{\frac{37}{35}+\frac{35}{35}}{4}$	
$\frac{-33}{35} \times \frac{1}{3} + \frac{\overset{18}{\cancel{72}}}{35} \times \frac{1}{\underset{1}{\cancel{4}}}$	
$\frac{-11}{35} + \frac{18}{35}$	
$\frac{7}{35}$	
$\frac{1}{5}$	

LS = RS

Do either the even-or the odd-numbered questions. If you need more practice, do all questions.

1. The speed of a car is $1\frac{2}{3}$ km/min. How long does it take for this car to travel a distance of 300 km? (Use the 4-step procedure.)

2. John walks to school every morning, a distance of 1 km. He can walk $66\frac{2}{3}$ m/min. How long does it take for him to walk to school? (Use the 4-step procedure.)

3. Find two consecutive numbers such that the sum of $\frac{1}{2}$ of the first number and $\frac{1}{3}$ of the second number is $\frac{7}{6}$. (Use the 4-step procedure.)

4. Find two consecutive numbers such that the difference of $\frac{1}{3}$ of the first number by $\frac{1}{4}$ of the second number is $\frac{1}{12}$. (Use the 4-step procedure.)

✓ For solutions to **Activity 3**, turn to the **Appendix, Topic 1**.

Solve and verify the following equations. (State the lowest common denominator for the fraction in each question and multiply each term by the common denominator.)

5. $\frac{2}{3}x=8$

6. $\frac{a}{5}=-3$

7. $\frac{x}{5}-\frac{x}{6}=10$

8. $\frac{x}{3}+\frac{x}{5}=4$

9. $\frac{x-1}{3}=\frac{x+2}{4}$

10. $\frac{2x-1}{3}=\frac{x}{5}$

11. $\frac{x+2}{3}-\frac{x-3}{4}=\frac{3}{2}$

12. $\frac{(x+5)}{3}-\frac{(x-1)}{2}=\frac{1}{6}$

✓ For solutions to Activity 3, turn to the **Appendix, Topic 1.**

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

The questions which follow review material that you learned in Mathematics 13. After you work through this section, take a second look at what you have studied in this topic.

Do all the questions. Fill in the blanks and check your answers.

1. Solve and verify $4x = 36$.
   Divide both sides by 4.

$$\frac{4x}{(\ )} = \frac{36}{(\ )}$$

Simplify. $x = (\ )$

Verify: $x = 9$

LS	RS
$4x$	36
$(\ )$	

LS = RS

2. Solve and verify $3y - 1 = 5$.
   Simplify. Add 1 to both sides.

$$3y - 1 + (\ ) = 5 + (\ )$$

$$3y = (\ )$$

Divide both sides by 3.

$$\frac{3y}{(\ )} = \frac{(\ )}{(\ )}$$

$$y = (\ )$$

Verify: $x =$

LS	RS
$3y - 1$	5

LS = RS

3. Solve and verify $2(x+1) + 3(x-7) = 6$.

Simplify.

$$2x + 2 + (\ )x - (\ \ ) = 6$$

$$(\ )x - (\ \ ) = 6$$

Add 19 to both sides.

$$(\ )x = (\ \ )$$

$$x = (\ \ )$$

$$x = (\ \ )$$

Verify: $x =$

LS	RS
$2(x + 1) + 3(x - 7)$	6

LS = RS

4. Solve and verify $3(x+1)-2(x-1)=7$.

Simplify. $3x+(\ )-2x+2=7$

Why is the 4th term a positive 2?

Simplify. $(\ )x+(\ )=7$

Complete. $(\ )x=(\ )$

$x=(\ )$

Verify: $x =$

LS	RS

LS = RS

For solutions to **Extra Help**, turn to the **Appendix, Topic 1.**

In solving linear equations with fractional coefficients, you must first clear the equation of fractions by multiplying both sides of the equation (multiply every term) by the lowest common denominator. Then the equation can be solved using one of the previous methods.

5. Solve and verify $\frac{3x}{4}-\frac{5x}{6}=\frac{1}{3}$.

The L.C.D. of 4, 6, and 3 is 12.

Multiply every term by 12. $\frac{3x}{4}(\ )-\frac{5x}{6}(\ )=\frac{1}{3}(\ )$

If you multiply every term by the lowest common denominator, you should be able to clear the equation of fractions.

Complete: $(\ )x-(\ )x=(\ )$

$(\ )x=(\ )$

$x=(\ )$

Verify: $x =$

LS	RS

LS = RS

For solutions to **Extra Help**, turn to the **Appendix, Topic 1.**

If you want more help, review the examples in this topic.

Now do the following questions.

Do a or b of each question which follows. If you need more practice, do both parts.

6. Solve: a. $3x = 81$ b. $7x = 56$

7. Solve: a. $5x - 3x + 8 = x + 11$

b. $7x + x - 3 = 2x + 13$

8. Solve: a. $\frac{x}{4} = 13$ b. $\frac{-x}{3} = 9$

9. Solve: a. $\frac{x}{4} + 3 = 13$ b. $\frac{x}{5} - 4 = -1$

10. Solve: a. $\frac{x}{5} + \frac{1}{2} = \frac{1}{3}$ b. $\frac{x}{2} - \frac{x}{4} = 3$

For solutions to **Extra Help**, turn to the **Appendix, Topic 1.**

Extensions

Take a look at another example.

Example 9

To raise money to purchase team jackets, the school volleyball teams sold team pennants. The pennants came in two sizes that sold for \$1.50 and \$2.25. The teams sold a total of 205 pennants for \$377.25. How many pennants of each size were sold?

Solution:

Step 1: Understand the problem.
Let x be the number of pennants sold for \$1.50 each.
Since there was a total 205 pennants sold, there must of been $205 - x$ pennants sold for \$2.25 each.
The total amount of money earned from the \$1.50 pennants will be $\$1.50x$.
The total amount of money earned from the \$2.25 pennants will be $\$2.25(205 - x)$.

Step 2: Develop a plan.
The sum of the two totals will be \$377.25. This can be easily translated into a mathematically equation.

$$\$1.50x + \$2.25(205 - x) = \$377.25$$

Step 3: Carry out the plan.
Now solve the equation for x and then $205 - x$.

$$
\begin{aligned}
\$1.50x + \$2.25(205 - x) &= \$377.25 \\
\$1.50x + \$461.25 - \$2.25x &= \$377.25 \\
\$461.25 - \$0.75x &= \$377.25 \\
-\$0.75x &= -\$84.00 \\
x &= \$112
\end{aligned}
$$

$$
\begin{aligned}
205 - x &= 205 - 112 \\
&= 93
\end{aligned}
$$

Step 4: Look back. Check the answer.

There were 112 pennants sold for \$1.50 × 112 or \$168.00.

There were 93 pennants sold for \$2.25 × 93 or \$209.25.

The total amount of the sales is
$\$168 + \$209.25 = \$377.25$.

This checks.

There were 112 pennants sold for \$1.50 each, and 93 pennants sold for \$2.25 each.

Try the following three questions.

1. Find two consecutive even numbers such that the difference between $\frac{1}{3}$ of the first number minus $\frac{1}{4}$ of the second number is $\frac{1}{3}$.
2. Find three consecutive numbers such that the sum of $\frac{1}{3}$ of the first number, $\frac{1}{5}$ of the second number, and $\frac{1}{2}$ the third number is 26.
3. Sara, Jack, and Ron form an investment group. They want to save some money so that they can invest their savings in a special project. Sara's monthly contribution is $\frac{1}{10}$ of his monthly income. Jack's contribution is $\frac{1}{2}$ of Sara's income less \$2.00. Ron's contribution is $\frac{3}{4}$ of Sara's income. The total monthly contribution is \$ 4048. What is Sara's monthly income?

For solutions to **Extensions**, turn to the **Appendix, Topic 1**.

Topic 2 Solving Linear Inequalities and Graphing Their Solutions

Introduction

Changing a key word in a sentence can also change its meaning. For example, "My friend has a dog." means something quite different from "My friend is a dog."

Likewise, changing a math symbol relating two expressions changes the meaning. When you change an = sign to > or <, you change an equation to an inequality which represents a completely different set of solutions.

In this topic, you will use your equation solving skills to solve inequations or inequalities and graph these inequalities to gain a better understanding of their meanings.

What Lies Ahead

Throughout the topic you will learn to

1. apply the Reverse the Sign Rule to solve and graph the solution of a linear inequality

Now that you know what to expect, turn to the next page to begin your study of solving linear inequalities and graphing their solutions.

Exploring Topic 2

Activity 1

Apply the Reverse the Sign Rule to solve and graph the solution of a linear inequality.

If the weights of two books are equal, then there is only one solution. if one book weighs 2N, then the other book also weighs 2N. If the weight of one book is greater than the weight of the second book, then there are an infinite number of possible solutions. If one book weighs 2N, then the weight of the second book can be anything greater than 2N. You would not be able to write down all the solutions. You have to use the > or < sign or a graph to represent the whole set of answers. An inequation or inequality is formed when the equal sign relating two expressions is replaced by one of the following inequality symbols.

< means **less than.**
> means **greater than.**
≤ means **less than or equal to.**
≥ means **greater than or equal to.**

Solving Inequalities

Before attempting to solve inequalities, find out whether the rules for solving equations can be used to solve inequalities. If you add, subtract, multiply, or divide both sides of an equation by the same positive or negative number, the two sides of the equation remain equal. What about an inequality? Consider the two numbers 6 and 8 in this inequality: $6 < 8$.

$$6 < 8$$

Operation	LS	RS	Is LS < RS?
Add a positive number (2) to both sides.	8	10	yes
Subtract a positive number (2) from both sides.	4	6	yes
Add a negative number (– 2) to both sides.	4	6	yes
Subtract a negative number (– 2) from both sides.	8	10	yes
Multiply both sides by a positive number (2).	12	16	yes
Multiply both sides by a negative number (– 2).	– 12	– 16	no
Divide both sides by a positive number (2).	3	4	yes
Divide both sides by a negative number (– 2).	– 3	– 4	no

These results suggest that all but two of the rules for solving equations apply to inequalities. When you multiply both sides by a negative number, the equation becomes false. Since $-12 > -16$, you know that the inequality sign must be reversed if you want to keep the original inequality true. When you divide both sides by a negative number, the same thing happens. You have to reverse the inequality sign in order to keep the inequality true. One should remember this important idea.

When multiplying or dividing both sides of an inequality by a negative number, you must reverse the inequality sign.

You are now ready to solve some inequalities.

Example 1

Solve $5x + 1 < 16$.

Solution:

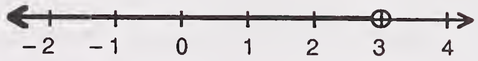
$5x+1<16$ Subtract 1 from both sides.

$5x<15$

$\frac{5x}{5}<\frac{15}{5}$ Divide both sides by 5.

$\therefore x<3$.

Pictures often are helpful. You can use a number line to represent this solution.



This number line is called the graph of inequalities.

How can you check this solution? Since the result states that a number less than 3 satisfies the inequality, all we need to do is choose a number less than 3 and test it. Suppose you choose 2.

$5x + 1 < 16$

LS	RS
$5x+1$	16
$5(2)+1$	
$10+1$	
11	

LS < RS

Since $11 < 16$, you have shown that when $x = 2$, LS < RS, which is the same as the original inequality.

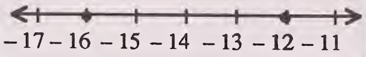
Now choose a number which is larger than 3. Try 4.

LS	RS
$5x+1$	16
$5(4)+1$	
21	

LS $\nless$ RS

It appears from these tests that the inequality is true for $x < 3$.

The hollow circle means the number 3 is not included in the solution.



-12 is further to the right.
$\therefore -12$ is greater than -16.

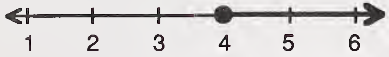
$\nless$ means not less than.

Example 2

Solve and graph $4x-3 \geq 5+2x$.

Solution:

$4x-3 \geq 5+2x$	
$4x-2x-3 \geq 5+2x-2x$	Subtract (2*x*) from both sides.
$2x-3 \geq 5$	Simplify.
$2x \geq 8$	Add 3 to both sides.
$x \geq 4$	Divide both sides by 2.



The solution indicates that any number greater than or equal to 4 satisfies the inequality. Verify the solution.

Verification:

Choose a number which is greater than 4 and prove that the left side of the inequality is greater than or equal to the right side. Let $x = 5$.

LS	RS
$4x-3$	$5+2x$
$4(5)-3$	$5+2(5)$
17	15

$\text{LS} \geq \text{RS}$

Choose a number which is less than 4 and prove that the left side of the inequality is **not** greater than or equal to the right side. Let $x = 3$.

LS	RS
$4x-3$	$5+2x$
$4(3)-3$	$5+2(3)$
9	11

$\text{LS} \ngeq \text{RS}$

It appears from these tests that the inequality is true for $x \geq 4$.

The next example is a trickier one. Make sure that you remember to use the Reverse the Sign Rule. When you multiply or divide both sides of an inequality by a negative number, you must reverse the inequality sign. < will become >, and > will become <.

The symbol $\ngeq$ means not greater than or equal to.

The solid dot at 4 indicates that the number 4 is part of the solution.

Example 3

Solve and graph $2(x-1)-7x < x+6$.

Solution:

$2x-2-7x < x+6$

$-2-5x < x+6$

$-5x-x < 6+2$

$-6x < 8$

$\frac{-6x}{-6} > \frac{8}{-6}$ Divide both sides by – 6 and reverse the inequality sign!

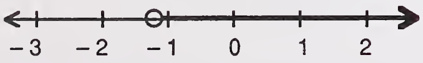
$x > -\frac{8}{6}$ Simplify.

$x > -\frac{4}{3}$

$x > -1\frac{1}{3}$

Therefore any number greater than $-1\frac{1}{3}$ satisfies the inequality.

Graph:



Verify $x = 1$.

LS	RS
$2(x-1)-7x$	$x+6$
$2(1-1)-7(1)$	$1+6$
-7	7

LS < RS

Verify $x = -2$.

LS	RS
$2(x-1)-7x$	$x+6$
$2(-2-1)-7(-2)$	$-2+6$
$2(-3)+14$	4
$-6+14$	
8	

LS ≮ RS

It appears from these tests that the inequality is true for $x > -1\frac{1}{3}$.

Now try one more example.

The hollow dot at $-1\frac{1}{3}$ indicates that $-1\frac{1}{3}$ is excluded from the solution.

Example 4

Solve and graph $-\frac{x}{2} \le -\frac{x}{3} - 6$.

Solution:
Multiply every term by 6.

$$\overset{3}{\cancel{6}}\left(-\frac{x}{\underset{1}{\cancel{2}}}\right) \le \overset{2}{\cancel{6}}\left(-\frac{x}{\underset{1}{\cancel{3}}}\right) - 6(6)$$

$$-3x \le -2x - 36$$

$$-3x + 2x \le -2x + 2x - 36$$

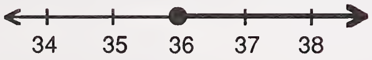
$$-x \le -36$$

$$\frac{-x}{-1} \ge \frac{-36}{-1}$$

$$x \ge 36$$

Since you are dividing by −1, the inequality must be reversed.

Graph:



Verify $x = 30$.

LS	RS
$-\frac{x}{2}$	$-\frac{30}{3} - 6$
$-\frac{30}{2}$	$-10 - 6$
-15	-16

LS $\not\le$ RS

Verify $x = 42$.

LS	RS
$-\frac{x}{2}$	$-\frac{42}{3} - 6$
$-\frac{42}{2}$	$-14 - 6$
-21	-20

LS $\le$ RS

It appears from these tests that the inequality is true for $x \ge 36$.

You should now be able to solve some inequalities on your own. If you feel you need more help before doing the questions, you may want to go to **Extra Help** first.

Notice how you are to use the Reverse the Sign Rule to keep the inequality true.

The solid dot at 36 indicates that 36 is included in the solution.

In each question, do a, d, or b, c.

1. Solve and graph each of the following inequalities. Verify your answer.

   a. $x-5\geq 11$
   b. $-3+5x<12$
   c. $3y-2>y+6$
   d. $y+3<2y-7$

2. Solve and graph each of the following inequalities.

   a. $3a<-15$

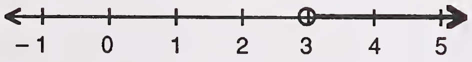
   b. $\frac{x}{7}\geq 2$

   c. $-3y>6$

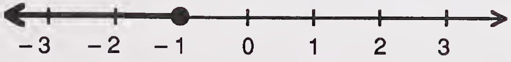
   d. $-\frac{2x}{3}\leq -2$

3. Write the inequality shown by each graph. Use x to represent the real number variable.

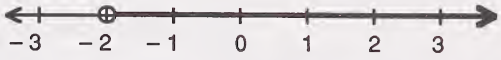
   a.



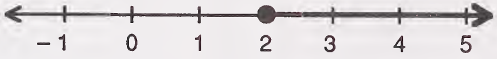
   b.



   c.



   d.



4. Solve the following inequalities.

   a. $2-3x\geq -19$

   b. $-\frac{1}{3}x+3<-9$

   c. $-3(2x-4)>4(x-1)-14$

   d. $5(x-3)<-2(x+2)+11$

✓ For solutions to **Activity 1**, turn to the **Appendix, Topic 2.**

If you want to try some harder questions, go to **Extensions**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.

Extra Help

If you find this topic difficult, do the following questions. Follow the order, follow each step, fill in the blanks, and answer all the questions.

1. Solve $x + 5 > 7$ and graph the solution.
   What is your first step?
   $x + 5 - (\ ) > 7 - (\ )$
   $x > (\ )$

   Graph:

   In your graph, did you use a solid dot or an open dot? ____________
   Why? ____

2. Solve $x - 5 \le 3$ and graph the solution.
   What is your first step?
   $x - 5 + (\ ) \le 3 + (\ )$
   $x \le (\ )$

   Graph:

   In your graph, did you use a solid dot or an open dot? ____________
   Why? ____

3. Solve $5x > 15$ and graph the solution.
   What is your first step?

   $$\frac{5x}{(\ )} > \frac{15}{(\ )}$$

   $x > (\ )$

   Do you have to reverse the inequality sign? ____________
   Why? ____

   Graph:

4. Solve $-5x > 15$ and graph the solution.
   What is your first step? ________________________________

   Insert inequality sign and fill in the blanks.

   $$\frac{-5x}{(\ )} \quad \frac{15}{(\ )}$$

   $x \quad (\ )$

   Do you have to reverse the inequality sign? ____________
   Why? ____

   Graph:

5. Solve $\frac{x}{3} \le 7$.

What is your first step? ____________________

Insert inequality sign and fill in the blanks.

$\frac{x}{3}(\ )\qquad 7(\ )$

$x \qquad (\ )$

Do you have to reverse the inequality sign? ____________

Why? ____________________

Graph:

6. Solve $\frac{x}{-3} \le 7$.

What is your first step? ____________________

Insert inequality sign and fill in the blanks.

$\frac{x}{-3}(\ )\qquad 7(\ )$

$x \qquad (\ )$

Do you have to reverse the inequality sign? ____________

Why? ____________________

Graph:

Now do the following. Do even-or odd-numbered questions. If you need more practice, do the rest of the questions.

7. Solve and graph $x + 3 < 5$.

8. Solve and graph $x - 4 \ge 3$.

9. Solve and graph $3x \ge 27$.

10. Solve and graph $-4x \le 12$.

11. Solve and graph $\frac{x}{4} > \frac{3}{2}$.

12. Solve and graph $\frac{-x}{3} < 2$.

✓ For solutions to **Extra Help**, turn to the **Appendix, Topic 2.**

Now you may want to review the examples in this topic.

Extensions

Dealing with fractions in inequations is the same as with equations. Find the lowest common denominator of the fractions and multiply this number through all of the terms. Examine how this was done in the following example.

$$\frac{5x+1}{12} \le \frac{4x+3}{10} - \frac{1}{4}$$

The L.C.D. is 60. Multiply 60 onto all three terms.

$$\overset{5}{\cancel{60}}\left(\frac{5x+1}{\underset{1}{\cancel{12}}}\right) \le \overset{6}{\cancel{60}}\left(\frac{4x+3}{\underset{1}{\cancel{10}}}\right) - \overset{15}{\cancel{60}}\left(\frac{1}{\underset{1}{\cancel{4}}}\right)$$

$$\begin{aligned}
5(5x+1) &\le 6(4x+3) - 15(1) \\
25x+5 &\le 24x+18-15 \\
25x+5 &\le 24x+3 \\
25x-24x+5 &\le 24x-24x+3 \\
x+5 &\le 3 \\
x+5-5 &\le 3-5 \\
x &\le -2
\end{aligned}$$

Check for $x = 2$.

LS	RS
$\frac{5x+1}{12}$	$\frac{4x+3}{10} - \frac{1}{4}$
$\frac{5(2)+1}{12}$	$\frac{4(2)+3}{10} - \frac{1}{4}$
$\frac{10+1}{12}$	$\frac{8+3}{10} - \frac{1}{4}$
$\frac{11}{12}$	$\frac{11}{10} - \frac{1}{4}$
$\frac{110}{120}$	$\frac{22}{20} - \frac{5}{20}$
	$\frac{17}{20}$
	$\frac{102}{120}$

$\text{LS} \not\le \text{RS}$

Check for $x = -4$.

LS	RS
$\frac{5x+1}{12}$	$\frac{4x+3}{10} - \frac{1}{4}$
$\frac{5(-4)+1}{12}$	$\frac{4(-4)+3}{10} - \frac{1}{4}$
$\frac{-20+1}{12}$	$\frac{-16+3}{10} - \frac{1}{4}$
$\frac{-19}{12}$	$\frac{-13}{10} - \frac{1}{4}$
$\frac{-190}{120}$	$\frac{-26}{20} - \frac{5}{20}$
	$\frac{-31}{20}$
	$\frac{-186}{120}$

$\text{LS} \le \text{RS}$

Therefore, $x \le -2$.

Now try the following equations.

1. Solve $\frac{x}{2} - \frac{2}{7}x + \frac{2}{3} \le \frac{1}{3}$.

2. Solve $x + \frac{1}{5}x - \frac{1}{2} > \frac{1}{10}$.

3. Solve for x when x and $(x + 1)$ are two consecutive real numbers such that the sum of the first and three times the second is larger than 15. Find x and draw a graph to represent the possible solution.

✓ For solutions to **Extensions**; turn to the **Appendix, Topic 2.**

Topic 3 Factoring Polynomials of the Form $ax^2 + bx + c;\ a, b, c, \in I;\ a \neq 0$

Introduction

Opening and closing, buttoning and unbuttoning, on and off, can be thought of as reverse operations because one undoes or reverses the other.

In algebra, factoring polynomials undoes multiplying polynomials. In this topic, you will learn some important factoring skills.

What Lies Ahead

Throughout the topic you will learn to

1. factor a trinomial of the form $ax^2 + bx + c$, where *a*, *b*, and *c* are integers
2. factor a perfect trinomial square
3. factor polynomials by applying more than one factoring method

Now that you know what to expect, turn to the next page and begin your study of factoring polynomials of the form $ax^2 + bx + c;\ a,\ b,\ c, \in I;\ a \neq 0$.

Exploring Topic 3

Activity 1

Factor a trinomial of the form ax^2+bx+c, where a, b, and c are integers, and $a \neq 0$.

Before you begin this topic, make sure that you remember how to factor a trinomial of the form x^2+bx+c, where the coefficient of x^2 is 1. If you forgot, then review Mathematics 13, Unit 2.

You have learned how to factor a trinomial where the coefficient of the first term is 1. In this topic, you are going to move one step further. You are going to factor trinomials where the coefficient of the first term is not 1.

Do you recall factoring the trinomial x^2+bx+c using binomial grids? This time, you have more than one x^2 and you are going to use the same method. Draw a rectangle that has all the x^2, x, and unit tiles in the trinomial. Make sure that all of the tiles are within the rectangle and that there are no extra tiles in the rectangle. The length and width of the rectangle are the factors of the trinomial. Look at the following example.

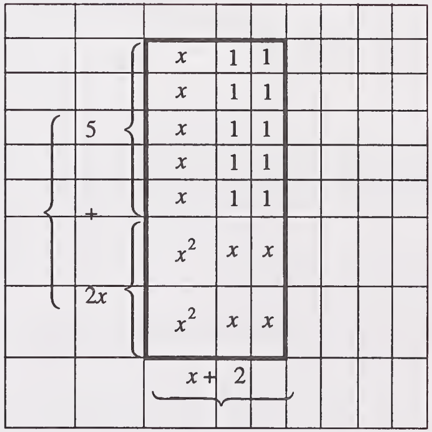
Example 1

Factor $2x^2+9x+10$.

Solution:

Draw a rectangle that has two x^2 tiles, nine x tiles, and ten unit tiles.

This represents a factor.



This represents another factor.

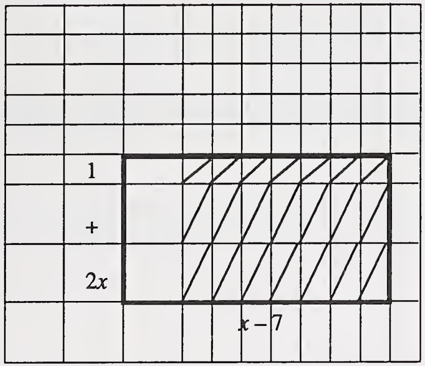
Count the number of tiles. Are the correct number of each tile in the rectangle with no extra tiles? Yes. Therefore, $2x^2+9x+10=(2x+5)(x+2)$.

If the middle term of the trinomial is negative, then use ▭ to represent a negative x tile. If the last term is negative, use ⧄ to represent negative unit tiles. Now look at another example.

Example 2

Factor $2x^2-13x-7$.

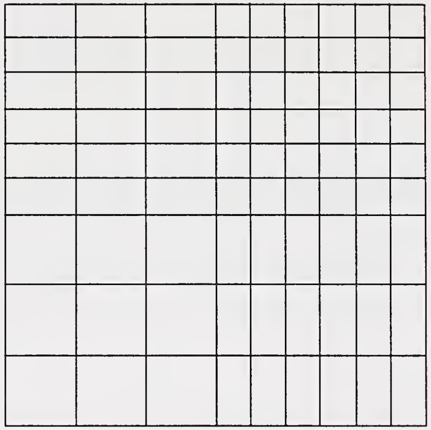
Solution:



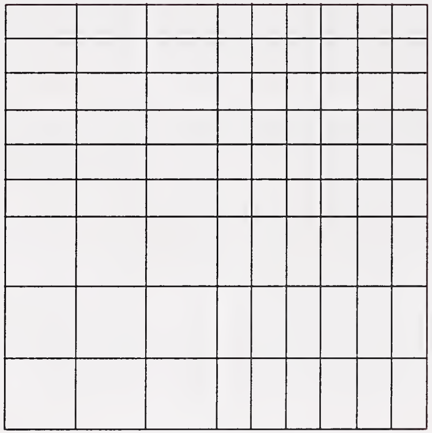
In the rectangle, there are 2 positive x^2 tiles, 14 negative x tiles, one positive x tile (a total of 13 negative x tiles), and 7 negative unit tiles. Therefore, $2x^2-13x-7=(2x+1)(x-7)$.

Do you want to try? Use the binomial grids provided to do the following questions.

1. Factor $2x^2+9x+4$.

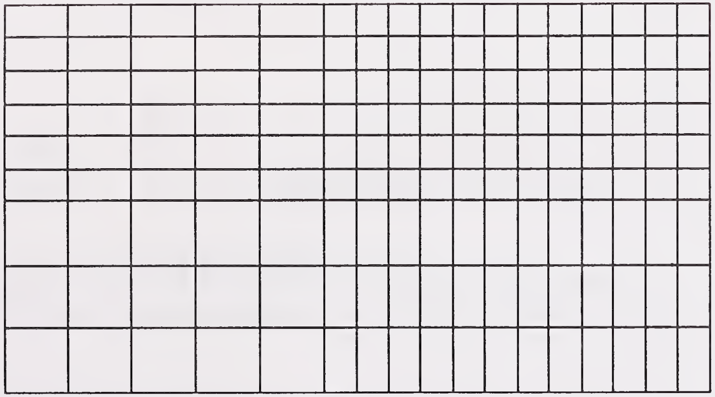


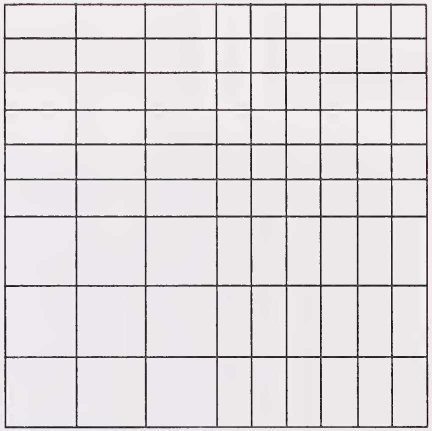
2. Factor $2x^2-13x+15$.



3. Factor $5x^2+7x-12$.

4. Factor $3x^2-19x-14$.





For solutions to **Activity 1**, turn to **Appendix, Topic 3**.

If you want more practice with binomial grid, factor questions 5, 6, 7, and 8 at the end of Activity 1 by using the binomial grid.

Now, if you want to use an algebraic method to factor a trinomial, you will have to experiment with possible factors using FOIL.

Use the following examples to explain the method used to factor a trinomial.

Case 1: If all three terms are positive, factor the trinomial as outlined in Example 3.

Example 3

Factor $2x^2 + 9x + 10$.

Solution:

Step 1: Find the possible factors of the 1st term: $(2x)\ (1x)$. Therefore, possible first terms of the binomial factors are $(2x \quad)\ (x \quad)$.

Step 2: Find the possible factors of the constant term: 1×10, 2×5. Possible 2nd terms of the binomial factors are the following:

$(2x \quad 1) \qquad (1x \quad 10)$
$(2x \quad 10) \qquad (1x \quad 1)$
$(2x \quad 2) \qquad (1x \quad 5)$
$(2x \quad 5) \qquad (1x \quad 2)$

Since all three terms of the trinomial are positive, the possible 2nd terms of the binomial factors are **all positive**. Although $(-2)\ (-5) = 10$, you don't need any negative 2nd term here because the middle term in the polynomial is positive.

Step 3: Check the middle terms.

For $(2x + 1)(x + 10)$, the middle term is $20x + x = 21x$.

For $(2x + 10)(x + 1)$, the middle term is $2x + 10x = 12x$.

For $(2x + 2)(x + 5)$, the middle term is $10x + 2x = 12x$.

For $(2x + 5)(x + 2)$, the middle term is $4x + 5x = 9x$.

Therefore, $2x^2 + 9x + 10 = (2x + 5)(x + 2)$.

In this example, did you notice that you didn't have to multiply out the two binomial factors?

Case 2: If the middle term is negative and the constant term is positive, follow Example 4.

Example 4

Factor $3x^2 - 7x + 2$.

Solution:

Step 1: The possible factors of the 1st term are $(3x)$ (x). Therefore, possible 1st terms of the binomial factors are $(3x \quad)$ $(x \quad)$.

Step 2: If the middle term of the trinominal is negative and the 3rd term is positive, the possible 2nd terms of the binomial factors must be **both negative**. Possible factors of the 3rd term are -1×-2.

Therefore, possible 2nd terms of the binomial factors are
$(3x - 1)$ $(x - 2)$ $(3x - 2)$ $(x - 1)$.

Step 3: Check the middle terms.

For $(3x - 1)(x - 2)$, the middle term is $-x - 6x = -7x$.

For $(3x - 2)(x - 1)$, the middle term is $-2x - 3x = -5x$.

Therefore, $3x^2 - 7x + 2 = (3x - 1)(x - 2)$.

Case 3: If the constant term is negative. (The middle term can be positive or negative.)

Example 5

Factor $3x^2 + 4x - 4$. The middle term is positive.

Solution:

Step 1: The possible factors of the 1st term are $(3x)$ (x). Therefore, possible 1st terms of the binomial factors are $(3x \quad)$ $(x \quad)$.

Step 2: Since the constant term is negative, one of the possible factors of the constant term must be negative and one must be positive. Possible factors of the 3rd term are $(4)(-1)$, $(-4)(1)$, or $(-2)(2)$.

Possible factors of the trinomial are

$(3x + 1)$ $(x - 4)$
$(3x - 1)$ $(x + 4)$
$(3x + 4)$ $(x - 1)$
$(3x - 4)$ $(x + 1)$
$(3x + 2)$ $(x - 2)$
$(3x - 2)$ $(x + 2)$

Step 3: Check the middle terms.

For $(3x+1)(x-4)$, the middle term is $x-12x=-11x$.

For $(3x-1)(x+4)$, the middle term is $-x+12x=11x$.

For $(3x+4)(x-1)$, the middle term is $-3x+4x=x$.

For $(3x-4)(x+1)$, the middle term is $3x-4x=-x$.

For $(3x+2)(x-2)$, the middle term is $2x-6x=-4x$.

For $(3x-2)(x+2)$, the middle term is $-2x+6x=4x$.

Therefore, $3x^2+4x-4=(3x-2)(x+2)$.

Look at another example which has a negative middle term.

Example 6

Factor $3x^2-x-4$.

Solution:

Step 1: Possible 1st terms of the binomial factors are:
$(3x \quad)(x \quad)$.

Step 2: One of the possible factors of the constant term must be negative; therefore, possible factors of the 3rd term are (4) (–1), (– 4) (1), or (– 2) (2). Possible binomial factors are

$(3x+1) \quad (x-4)$
$(3x-1) \quad (x+4)$
$(3x+4) \quad (x-1)$
$(3x-4) \quad (x+1)$
$(3x+2) \quad (x-2)$
$(3x-2) \quad (x+2)$

Step 3: Check the middle terms.

For $(3x+1)(x-4)$, the middle term is $x-12x=-11x$.

For $(3x-1)(x+4)$, the middle term is $-x+12x=11x$.

For $(3x+4)(x-1)$, the middle term is $-3x+4x=+x$.

For $(3x-4)(x+1)$, the middle term is $-4x+3x=-x$.

This one is correct because it has the factors which give $-x$ as the middle term.

Therefore, $3x^2-x-4=(3x-4)(x+1)$.

You do not have to check all the possible factors, just check possible factors till you come up with the correct factors.

Are the above examples very long? With practice, you can combine the steps and make the whole procedure shorter. It is not necessary to write all the steps. Your ability to choose the correct factors quickly will improve with practice.

Now do questions 5 to 8 using the algebraic method to factor the trinomials. If you need more help, go to **Extra Help**.

5. Factor $6x^2+5x+1$.
6. Factor $6x^2-11x+3$.
7. Factor $3x^2+7x-6$.
8. Factor $15x^2-x-2$.

For solutions to **Activity 1**, turn to the **Appendix, Topic 3.**

If you want more practice with the algebraic method factor questions 1, 2, 3, and 4 in the binomial grid section using this method.

Activity 2

Factor a perfect trinomial square.

Can you see a pattern in the following examples?

$$\begin{aligned}(x+1)^2 &= (x+1)(x+1)\\ &= x^2+x+x+1\\ &= x^2+2x+1\end{aligned}$$

$$\begin{aligned}(2x-1)^2 &= (2x-1)(2x-1)\\ &= 4x^2-2x-2x+1\\ &= 4x^2-4x+1\end{aligned}$$

$$\begin{aligned}(3x-2)^2 &= (3x-2)(3x-2)\\ &= 9x^2-6x-6x+4\\ &= 9x^2-12x+4\end{aligned}$$

In each case, the first and last term of the trinomial are perfect squares and they are positive. The middle term is always twice the product of the two binomial terms and the sign of the middle term is the same as the sign of the binomial factor. If you can identify a perfect square trinomial, you can tell what the two identical binomial factors are. Now try one.

In a perfect trinomial square, the first and last terms are perfect squares and the middle term is twice the product of the square roots of the first and last terms.

Example 7

Factor $9x^2+30x+25$.

Solution:

The first term of the trinomial is $9x^2$ which is a perfect square $(3x)^2$. The third term of the trinomial is 25 which is a perfect square $(5)^2$. The middle term of the trinomial is $30x$, and $30x = 2(3x)(5)$.
Therefore, $9x^2+30x+25$ is a perfect trinomial square and $9x^2+30x+25=(3x+5)(3x+5)$

$$=(3x+5)^2.$$

Now try to factor a trinomial that has a negative middle term.

Example 8

Factor $4x^2-28x+49$.

Solution:

The first term $\left(4x^2\right)$ is a perfect square $(2x)^2$.
The last term (49) is a perfect square $(7)^2$.
The middle term is negative and $28x = 2(2x)(7)$.

Therefore, $4x^2-28x+49=(2x-7)(2x-7)$

$$=(2x-7)^2.$$

Do any three of the following four questions.

1. Factor $16x^2+24x+9$.
2. Factor $25x^2+70x+49$.
3. Factor $36x^2-60x+25$.
4. Factor $9x^2-6x+1$.

✓ For solutions to **Activity 2**, turn to the **Appendix, Topic 3.**

If you can identify a perfect trinomial square, you don't have to write these 3 steps. With practice it is not difficult to figure out the answer by observation.

The middle term is positive. Therefore, the sign in the binomial factors is also positive.

Activity 3

Factor trinomials by applying more than one factoring method.

Sometimes it is not that easy to identify a polynomial, and you may have to use more than one method to factor a polynomial. Keep in mind that the first thing is to look for factors common to every term, then try to factor the remaining polynomial if possible.

Example 9

Factor $2x^3-20x^2+50x$.

Solution:

$$\begin{aligned}2x^3-20x^2+50x &= 2x\left(x^2-10x+25\right)\\ &= 2x(x-5)^2\end{aligned}$$

Now look at a harder example.

Example 10

Factor $\left(x^2-4x+4\right)-\left(9y^2+6y+1\right)$.

Solution:

$$\left(x^2-4x+4\right)-\left(9y^2+6y+1\right)$$

$$=(x-2)^2-(3y+1)^2$$

$$=\left[(x-2)+(3y+1)\right]\left[(x-2)-(3y+1)\right]$$

$$=(x+3y-1)(x-3y-3)$$

Each is a perfect square.

This is a difference of two squares.

Try some factoring on your own.
Do a or b of each question. If you need more practice, do the rest of the questions.

1. Factor:
   - a. $5x^2+35x+50$
   - b. $2x^2-2x-112$
2. Factor:
   - a. $3y^3-15y^2-42y$
   - b. $5y^3-60y^2+180y$
3. Factor:
   - a. $\left(4a^2+12a+9\right)-\left(b^2-10b+25\right)$
   - b. $\left(4x^2-8x+4\right)-\left(y^2+4y+4\right)$

For solutions to **Activity 3** turn to the **Appendix, Topic 3.**

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

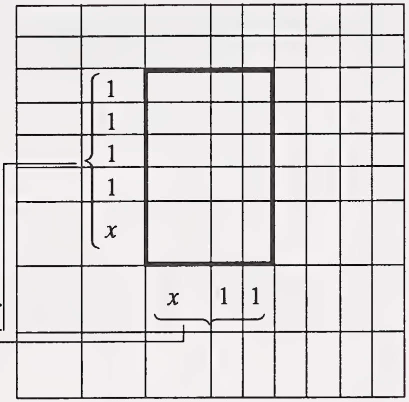
} You may decide to do both.

Extra Help

Do you remember how to factor a trinomial $ax^2 + bx + c$ where $a = 1$ and $b, c \in I$. If you don't, do the following exercises step by step.

In Mathematics 13, you used a binomial grid to factor a trinomial $x^2 + bx + c$. x^2 tiles are the larger squares. x tiles are the rectangles. Unit tiles are the small squares.

To factor a trinomial like $x^2 + 6x + 8$, you start from the x^2 tile and draw a rectangle which has $1x^2$ tile, 6 x tiles and 8 unit tiles.



Therefore, the factors are

$$x^2 + 6x + 8 = (x + 2)(x + 4).$$

If the coefficient of x^2 is not 1, then your rectangle would have to cover more than $1x^2$ tile.

Now use the algebraic method to factor a trinomial. This method is completely different from the one you have learned. Some people may find this method easier. If you find the other algebraic method difficult, then try this one. This method is called the **decomposition method.**

Example 11

Use the decomposition method to factor $3x^2 + 20x + 12$.

Solution:

Step 1: Multiply the coefficient of x^2 by the third term.

$$3 \times 12 = 36$$

Step 2: Decompose the coefficient of the middle term $20x$ into two numbers that multiply to +36 and add to +20. These numbers are 18 and 2. Express the middle terms as $18x + 2x$.

$$3x^2 + 20x + 12 = 3x^2 + 18x + 2x + 12$$

Step 3: Group the first two terms and the last two terms in pairs to remove common factors.

$$\left(3x^2 + 18x\right) + (2x + 12)$$

$$= 3x(x + 6) + 2(x + 6) \quad \text{[Take out the common factor } (x + 6).]$$

$$= (x + 6)(3x + 2)$$

Step 4: Check the middle term.

$$(x+6)(3x+2)=18x+2x$$
$$=20x$$

Therefore, $3x^2+20x+12=(x+6)(3x+2)$.

If the middle of the trinomial is negative and the third term is positive, then the two numbers in Step 2 must be both negative. Try an example of this type of trinomial.

Example 12

Factor $3x^2-7x+2$.

Solution:

Step 1: The product of the 1st term coefficient and the constant term is $3\times 2=6$.

Step 2: Decompose the coefficient of the middle term.
$-7x=-6x-x$ because
$(-6)(-1)=6$ and $(-6)+(-1)=-7$.
$3x^2-7x+2=3x^2-6x-x+2$

Step 3: Group the 1st two terms and the last two terms in pairs to remove common factors.

$$\left(3x^2-6x\right)+(-x+2)$$
$$=3x(x-2)-1(x-2)$$
$$=(x-2)(3x-1)$$

−1 is factored out of the second pair so that both factors are $(x-2)$.

Step 4: Check the middle term.

$$(x-2)(3x-1) \quad : \quad -x-6x=-7x$$

Therefore, $3x^2-7x+2=(x-2)(3x-1)$.

If the last term of the trinomial is negative, then one of the two numbers in Step 2 must be negative and the other one positive. Now try an example of this type of trinomial.

Example 13

Factor $3x^2-5x-2$.

Solution:

Step 1: The product of the 1st coefficient and the constant term is $3\times(-2)=-6$.

Step 2: Decompose the coefficient of the middle term
$-5x=-6x+x$ because
$(-6)+(1)=-5$ and $(-6)(1)=-6$.
$3x^2-5x-2=3x^2-6x+x-2$

Step 3: Group the 1st two terms and the last two terms in pairs to remove common factors.

$$\left(3x^2-6x\right)+1(x-2)$$
$$=3x(x-2)+1(x-2)$$
$$=(x-2)(3x+1)$$

Step 4: Check the middle term.

$$(x-2)(3x+1) \quad : \quad x-6x=-5x$$

Therefore, $3x^2-5x-2=(x-2)(3x+1)$.

It is not necessary to show your solution step by step like the examples. You may want to combine the steps and make your solution shorter. Your ability to decompose the middle term will improve with practice.

Now do any two of the following four questions. If you want more practice, do them all.

1. Factor $10x^2+11x+3$.
2. Factor $10x^2-11x+3$.
3. Factor $6x^2-7x-5$.
4. Factor $6x^2+7x-5$.

For solutions to **Extra Help** turn to the **Appendix, Topic 3**.

Extensions

For some polynomials, once you have obtained the factors, you will notice that these factors can be factored again. Factoring a polynomial more than once can be a very challenging proposition. To crack these cases, you have to try a number of different approaches until you find the method that is successful. With a little experience, you will develop an inner understanding that will lead to the best method. Take a look at the following examples and note the clues that are being used to find the factors.

Example 14

Factor $15pq+25p^2-10pq^2-6q^3$.

Solution:

Clue 1. The first thing to always check for is a common factor. Examining this polynomial you will notice that there is no common factor.

Clue 2. Try to group three of the terms into a trinomial that can be factored. Once again, this does not help.

Clue 3. Group the terms into pairs and divide out any common factors. This will work.

$15pq+25p^2-10pq^2-6q^3$

$=\left(15pq+25p^2\right)+\left(-10pq^2-6q^3\right)$

The first pair has a common factor of $5p$.

$=5p(3q+5p)+\left(-10pq^2-6q^3\right)$

The second pair has a common factor of $-2q^2$.

$=5p(3q+5p)-2q^2(5p+3q)$

Are you done? Not yet.
Notice that both pairs have the factor $(3q + 5p)$.
This is now the common factor.

$=5p(3q+5p)-2q^2(3q+5p)$

$=(3q+5p)\left(5p-2q^2\right)$

The polynomial is now factored.

Example 15

Factor y^4-5y^2+4.

Solution:

Clue 1. First you will notice that this is a trinomial and it can easily be factored.

y^4-5y^2+4

$=\left(y^2-4\right)\left(y^2-1\right)$

Could it be this easy? Not quite.
Examine the two factors, do you notice anything particular about them? They are both difference of squares. Both of these factors can be factored again.

$=(y+2)(y-2)(y+1)(y-1)$

Now the polynomial is factored.

Do any of the following 3 questions.

1. Factor $30x^2y-80xy+40y$.

2. Factor $1-x^2+4x-4$.

3. Find the perimeter of the square if its area is $x^2-14x+49$ square units.

For the solution to **Extensions**, turn to the **Appendix, Topic 3.**

Topic 4 Solving Simple Quadratic Equations

Introduction

"It's time to get dramatic.
As we study the quadratic."

In a previous topic, you reviewed how to solve a linear or a first degree equation. The next logical step should be to investigate the solution of a second degree equation.

About 2000 B.C., ancient Egyptians and Babylonians were the first people known to use quadratic equations in their problem solving. Various methods have been used to solve these equations.

Take a look at how to solve quadratic equations.

What Lies Ahead

Throughout the topic you will learn to

1. solve and verify simple quadratic equations by reducing to $x^2 = c,\ c > 0$
2. solve and verify simple quadratic equations by factoring

Now that you know what to expect, turn to the next page and begin your study of solving simple quadratic equations.

Exploring Topic 4

Activity 1

Solve and verify simple quadratic equations by reducing to $x^2 = c,\ c > 0$.

If the area of a square is 4 cm^2, what are the dimensions of the square? To answer this question, you use x to represent the length of the side, x^2 to represent the area of the square and you have the equation $x^2 = 4$.

This is an equation of degree **2** and it is called a **quadratic equation.** $x^2 = 4$ is the simplest kind of quadratic equation. To solve this equation, you simply find the square root of both sides of the equation.

$$\begin{aligned} x^2 &= 4 \\ \sqrt{x^2} &= \pm\sqrt{4} \\ x &= \pm 2 \\ x &= 2 \quad \text{or} \quad x = -2 \end{aligned}$$

Since the length of a side cannot be negative, you only use the positive dimension. The dimensions of the square are 2 cm × 2 cm. Note that the number on the right side of the equation must be positive; otherwise, you don't have a solution because the square root of a negative number is undefined. If the question is not a word problem, make sure you remember that every number has two square roots.

Remember $\sqrt{\ }$ undoes squaring.

Note: You must consider both the positive and negative solutions when taking the square root of both sides of an equation.

If you have trouble with square roots, review Unit 1.

Example 1

Solve and verify $x^2 = 25$.

Solution:

$$\sqrt{x^2} = \pm\sqrt{25}$$

$$x = \pm 5$$

To make sure that the solutions you have obtained are correct, you should verify the answers in the original equation.

Verify $x = 5$.

LS	RS
x^2	25
5^2	
25	

LS = RS

Verify $x = -5$.

LS	RS
x^2	25
$(-5)^2$	
25	

LS = RS

Therefore, both solutions are correct.

This type of quadratic equation can be solved in an alternate way.

If you move the number to the left hand side of the equation, then x^2 and the numbers form a difference of two squares which you can factor. After you factor the binomial, make each factor equal to zero and solve each factor for the unknown variable. Examine how it works in the next example.

Example 2

Solve and verify $x^2 = 9$.

Solution:

$$x^2 = 9$$

$$x^2 - 9 = 0$$

$$x^2 - 3^2 = 0$$

$$(x+3)(x-3) = 0$$

(Use the zero product property.)

$$x + 3 = 0 \qquad x - 3 = 0$$

$$x = -3 \qquad x = 3$$

Verify $x = -3$.

LS	RS
x^2	9
$(-3)^2$	
9	

LS = RS

Verify $x = 3$

LS	RS
x^2	9
3^2	
9	

LS = RS

If the constant in the equation is not a perfect square, use the method shown in Example 1. The solution can either be written in radical form or an approximation of the solution can be found using your calculator.

Zero Product Property: If $ab = 0$, then either $a = 0$ or $b = 0$.

The following example shows you how you can factor a quadratic binomial where the number on the right hand side is not a perfect square.

Example 3

Solve and verify $x^2 = 3$.

Solution:

$$\begin{aligned} x^2 &= 3 \\ x^2 - 3 &= 0 \\ x^2 - \left(\sqrt{3}\right)^2 &= 0 \\ \left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right) &= 0 \end{aligned}$$

$x + \sqrt{3} = 0 \qquad x - \sqrt{3} = 0$

$x = -\sqrt{3} \qquad x = \sqrt{3}$

Verify $x = -\sqrt{3}$.

LS	RS
x^2	3
$\left(-\sqrt{3}\right)^2$	
3	

LS = RS

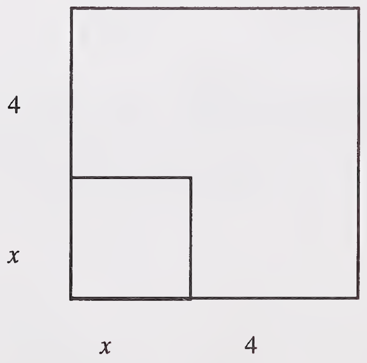
Verify $x = \sqrt{3}$.

LS	RS
x^2	3
$\left(\sqrt{3}\right)^2$	
3	

LS = RS

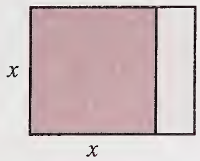
Now it is time for you to try some quadratic equations. Do either the odd-or the even-numbered questions. Do them all if you need the practice.

1. Solve and verify $x^2 = 16$.
2. Solve and verify $x^2 = 64$.
3. Solve and verify $3x^2 = 75$.
4. Solve and verify $7x^2 = 343$.
5. If the area of a square is 121 cm^2, what are the dimensions of this square?
6. Solve and verify $x^2 = 21$.
   (Give your answer to three decimal places.)
7. The area of the large square is 36 cm^2.
   Find x .



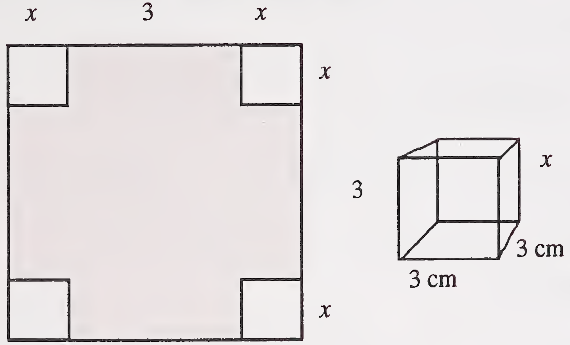
In Example 3, you can leave your answer in radical form. You may have to use your calculator if you are asked to give the answer without radicals.

8. In the figure below, the area of the large rectangle (square and small rectangle) is 210 cm ^{2} and the small rectangle is 41 cm ^{2}. Find the dimensions of the square.



9. Solve and verify $x^2 = 123$.
(Round your answer to two decimal places.)

10. A lidless square box has a base of 3 cm × 3 cm. The box is made by folding a piece of square cardboard. Equal squares are cut from the four corners. The area of the cardboard is 225 cm ^{2}. Find the height of the box.



For solutions to **Activity 1**, turn to the **Appendix, Topic 4.**

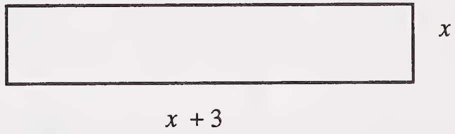
Activity 2

Solve and verify simple quadratic equations by factoring.

The area of a rectangle is 10 cm ^{2} and the length of the rectangle is 3 cm longer than the width. How do you find dimensions of the rectangle? To answer this question, follow the 4-step approach.

Step 1: Understand the problem.

Draw a diagram and define the unknowns.



Let x = the width of the rectangle.
$x + 3$ = the length of the rectangle.

Step 2: Develop a plan.

Since Area = length × width,
$10 = (x)\ (x + 3)$.

Step 3: Carry out the plan.
Simplify $10 = x(x+3)$.

$$10 = x^2 + 3x$$

$$x^2 + 3x - 10 = 0$$

This is an equation of degree 2; therefore, it is a quadratic equation. To solve a quadratic equation, you can try to transform a quadratic equation into the product of two linear factors. After you factor the quadratic equation, you can apply the zero product property and set each factor equal to zero (See Example 2.) and find the solutions. Then you must verify the solutions.

$$x^2+3x-10=0$$
$$(x+5)(x-2)=0$$
$$\begin{array}{rcl} x+5=0 & \quad & x-2=0 \\ x=-5 & & x=2 \end{array}$$

Since x cannot be negative, $x = 2$, and $x + 3 = 5$.

Step 4: Look back. Check your answer.

Verify $x = 2$.

$$\begin{array}{r|l} \text{LS} & \text{RS} \\ \hline x^2+3x-10 & 0 \\ 2^2+3(2)-10 & \\ 4+6-10 & \\ 0 & \end{array}$$
$$\text{LS} = \text{RS}$$

Therefore, the dimensions of the rectangle are 2 cm by 5 cm.

In Mathematics 23, all the quadratic equations you have to solve are quadratic equations which can be changed into the product of two linear factors. There are quadratic equations which you cannot factor. You will learn how to solve this kind of quadratic equation in Mathematics 33. Now, solve another quadratic equation.

Example 4

Solve and verify $6x^2-13x+5=0$.

Solution:

Factor $(2x-1)(3x-5)=0$.

Set each linear factor equal to zero.
$$2x-1=0 \quad \text{or} \quad 3x-5=0$$

Solve: $2x = 1$ $\qquad$ $3x = 5$

$$x=\frac{1}{2} \qquad x=\frac{5}{3}$$

Verify $x=\frac{1}{2}$.

$$\begin{array}{r|l} \text{LS} & \text{RS} \\ \hline 6x^2-13x+5 & 0 \\ 6\left(\frac{1}{2}\right)^2-13\left(\frac{1}{2}\right)+5 & \\ \frac{3}{2}-\frac{13}{2}+5 & \\ 0 & \end{array}$$
$$\text{LS} = \text{RS}$$

Linear means degree 1.

The zero product property states that if $ab = 0$, then either $a = 0$ or $b = 0$.

Verify $x = \frac{5}{3}$.

LS	RS
$6x^2 - 13x + 5$	0
$6\left(\frac{5}{3}\right)^2 - 13\left(\frac{5}{3}\right) + 5$	
$\frac{50}{3} - \frac{65}{3} + \frac{15}{3}$	
0	

LS = RS

Now try the following questions.

In 1 to 3, do either a or b. Then do either 4 and 6, or 5 and 7. If you need more practice, do the rest of the questions.

1. Solve and verify:

   a. $x^2 - 3x - 28 = 0$ b. $x^2 - 9x - 36 = 0$

2. Solve and verify:

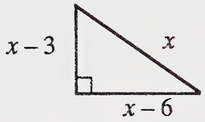
   a. $2x^2 + 9x - 35 = 0$ b. $6x^2 + 11x - 35 = 0$

3. Solve and verify:

   a. $10x^2 + 17x + 3 = 0$ b. $20x^2 + 39x + 7 = 0$

4. One number is 2 more than the other and they are both positive. The product of the 2 numbers is 15. Find the two numbers.

5. An x m ladder is leaning against a wall as shown in the diagram below.



The length of the sides of the right triangle are related by

$$x^2 = (x-3)^2 + (x-6)^2.$$

Solve for x .

6. The length of a rectangle is 5 cm longer than the width. The area of the rectangle is 24 cm ^{2}. Find the length of the rectangle.

7. One number is 7 less than the other and they are both positive. The product of the two numbers is 18. Find the two numbers.

✓ For solutions to **Activity 2**, turn to the **Appendix, Topic 4.**

If you require help, do the Extra Help section.
If you want more challenging explorations, do the Extensions section.
} You may decide to do both.

Extra Help

To solve a quadratic equation, you always write the equation so that one side is equal to zero. Then you try to factor the quadratic expression. If a quadratic equation is written in the form $x^2 = a$, it is easier to solve by simply taking the square roots of both sides of the equation. First make sure that you remember the plus and minus signs. To factor the difference of two squares is just an alternative method. If you don't remember how the difference of two squares works, review this concept again.

Since $(a+b)(a-b) = a^2 - b^2$,

$$a^2 - b^2 = (a+b)(a-b).$$

$a^2 - b^2$ is the difference of the two squares a^2 and b^2. To factor a difference of two squares, you follow these steps.

1. Express each term as a square of its square root.
2. Set up two binomials using each square root as a term in each binomial, with one binomial as the sum of the two square roots and the other binomial as the difference of the two square roots.
3. Check your work by multiplying your factors to make sure you obtain the original binomial.

Look at the following examples.

Example 5

Factor $9x^2 - 25$.

Solution:

$$\begin{aligned} 9x^2 - 25 &= (3x)^2 - (5)^2 \\ &= \underset{\underset{\text{Sum}}{\uparrow}}{(3x+5)}\underset{\underset{\text{Difference}}{\uparrow}}{(3x-5)} \end{aligned}$$

Now you could go on to solve for x.

Example 6

Factor $(y-3)^2 - 64$.

Solution:

$$\begin{aligned} (y-3)^2 - 64 &= (y-3)^2 - 8^2 && \leftarrow \text{Step 1} \\ &= [(y-3)\underset{\underset{\text{Sum}}{\uparrow}}{+}8][(y-3)\underset{\underset{\text{Difference}}{\uparrow}}{-}8] && \leftarrow \text{Step 2} \\ &= (y+5)(y-11) \end{aligned}$$

Now you could go on to solve for the possible values of y by using the zero product property.

Now do the following.

Do a or b of each question.

1. Factor:

   a. $81-4y^2$ b. $9-4x^2$

2. Factor:

   a. $1-(x+3)^2$ b. $(3y+2)^2-1$

3. Factor:

   a. $18x^2-50$ b. $72-2y^2$

4. Factor :

   a. $(x+1)^2-(x-1)^2$ b. $(x-3)^2-(x+1)^2$

For solutions to **Extra Help,** turn to the **Appendix, Topic 4.**

Extensions

Sometimes the difference of two squares can involve more than one variable. When you factor the difference of such squares, the same procedures apply.

Example 7

Factor $9ab^2-4a^3$.

Solution:

$$\begin{aligned} 9ab^2-4a^3 &= a\left(9b^2-4a^2\right) && \text{Divide out common factor.} \\ &= a\left[(3b)^2-(2a)^2\right] && \text{Write as squares.} \\ &= a(3b+2a)(3b-2a) && \text{Factor by difference of squares.} \end{aligned}$$

Example 8

Factor $18x^2y^3-50yz^4$.

Solution:

$$\begin{aligned} 18x^2y^3-50yz^4 &= 2y\left[9x^2y^2-25z^4\right] && \text{Divide out common factor.} \\ &= 2y\left[(3xy)^2-\left(5z^2\right)^2\right] && \text{Write as squares.} \\ &= 2y\left(3xy+5z^2\right)\left(3xy-5z^2\right) && \text{Factor by difference of squares.} \end{aligned}$$

Factoring Trinomials

A trinomial can involve more than one variable. The same method can be used to factor any trinomial. Look at the following examples.

Example 9

Factor $3x^3-15x^2y+12xy^2$.

Solution:
Divide out the common factor.

$$3x^3-15x^2y+12xy^2=3x\left(x^2-5xy+4y^2\right)$$

Factor the trinomial.
Possible 1st term of the binomial factors are $(x\quad)\ (x\quad)$.
Possible 2nd term of the binomial factors are $(\quad -y)\ (\quad -4y)$
or $(\quad -2y)\ (\quad -2y)$.
Thus, the possible binomial factors are $(x-y)(x-4y)$
or $(x-2y)(x-2y)$.

Check the middle term.

For $(x\ -\overbrace{y)(x}-4y)$, the middle term is $-4xy-xy=-5xy$.

Those are the correct binomial factors.
Therefore, $3x\left(x^2-5xy+4y^2\right)=3x(x-y)(x-4y)$.

Example 10

Factor $x^2+6xy+9y^2$.

Solution: This is a perfect square trinomial because

$$\begin{aligned}x^2+6xy+9y^2&=x^2+2(x)(3y)+(3y)^2\\&=(x+3y)^2\end{aligned}$$

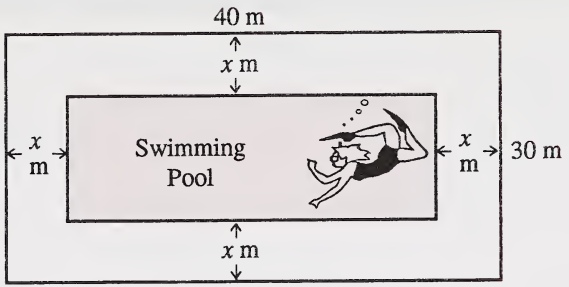
It is time for you to try some factoring. Do questions 1 to 4. If you want some harder questions, do 5 to 7.

Do a or b of each question. If you need more practice, do the rest of the questions.

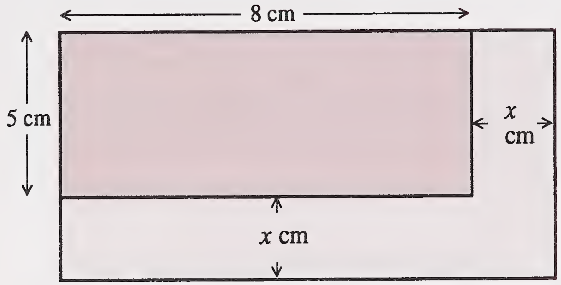
1. Factor:
   a. $49x^2-25y^2$ b. x^2-16y^2
2. Factor:
   a. $27mn^4-12m^3$ b. $8x^3y^2-2x$
3. Factor:
   a. $15x^2y+40xy^2+20y^3$ b. $6x^2y+16xy^2+10y^3$
4. Factor:
   a. $x^2-14xy+49y^2$ b. $18x^2-60xy+50y^2$
5. Solve and verify:
   a. $\frac{x+1}{2}+\frac{1}{x}=x+1$ b. $\frac{x-2}{3}+\frac{1}{2x}=x-\frac{1}{2}$
6. a. Find two consecutive numbers if the sum of the square of the larger number and three times the smaller number is 37.

   b. Find two consecutive numbers if the difference of the square of the larger number by two times the smaller number is 2.

7. a. A swimming pool is surrounded by a rectangular lawn as show below. If the total area of the lawn is equal to the area of the swimming pool, find the dimensions of the swimming pool.



b. A 5 cm × 8 cm rectangle is extended by x cm in each direction. The area of the new rectangle is 70 cm^2. Find the new dimensions.



For solutions to **Extensions**, turn to the **Appendix, Topic 4.**

Unit Summary

What You Have Learned

Your work with linear equations previous to Mathematics 23 was limited to linear equations with integral coefficients. In this unit you learned how to solve linear equations with rational coefficients. Solving inequalities was something new. In this unit, you also learned how to factor a trinomial of the form $ax^2 + bx + 3$ where a could be any integer larger than 1. Quadratic equations and the applications of quadratic equations are very important and useful. You learned how to solve quadratic equations in this unit. These skills will be useful as you use algebra to solve new types of problems in future mathematics courses.

You are now ready to

complete the **Unit Assignment**.

Appendix

Solutions

Review

Topic 1	**Solving and Verifying Linear Equations**
Topic 2	**Solving Linear Inequalities and Graphing Their Solutions**
Topic 3	**Factoring Polynomials of the Form** $ax^2+bx+c;\ a, b, c \in I$
Topic 4	**Solving Simple Quadratic Equations**

Review

1. a. $\frac{3}{\underset{1}{\cancel{5}}} \times \overset{15}{\cancel{75}} = 45$ b. $\frac{11}{\underset{1}{\cancel{9}}} \times \overset{5}{\cancel{45}} = 55$

2. 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102, 108, 114, 120, 126, 132, 138, 144, 150, 156, 162, 168, 174, 180, 186, 192, 198, 204, 210, . . .

   10: 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, 160, 170, 180, 190, 200, 210, 220, . . .

   14: 14, 28, 42, 56, 70, 84, 98, 112, 126, 140, 154, 168, 182, 196, 210, 224, . . .

   L.C.D. = 210

3. a. $46 = 2 \times 23$ b. $63 = 3 \times 3 \times 7$

4. a.
$$\begin{aligned} 36 &= 2 \times 2 \times 3 \times 3 \\ 54 &= 2 \times 3 \times 3 \times 3 \\ \text{G.C.F.} &= 2 \times 3 \times 3 \\ &= 18 \end{aligned}$$

   b.
$$\begin{aligned} 9x^2y^3 &= 3 \times 3 \times x \times x \times y \times y \times y \\ 27xy^2 &= 3 \times 3 \times 3 \times x \times y \times y \\ \text{G.C.F.} &= 3 \times 3 \times x \times y \times y \\ &= 9xy^2 \end{aligned}$$

5. a.
$$\begin{aligned} 7^2 \times 7^5 &= 7^{2+5} \\ &= 7^7 \end{aligned}$$

   b.
$$\begin{aligned} x \times x^3 \times x^7 &= x^{1+3+7} \\ &= x^{11} \end{aligned}$$

   c.
$$\begin{aligned} \frac{-28x^4y^3}{7x^2y} &= -4x^{4-2}y^{3-1} \\ &= -4x^2y^2 \end{aligned}$$

6. a.
$$\begin{aligned} 4x &= 16 \\ x &= 4 \end{aligned}$$

   b.
$$\begin{aligned} x + 11 &= 37 \\ x &= 37 - 11 \\ x &= 26 \end{aligned}$$

   c.
$$\begin{aligned} 3x - 2 &= 14 \\ 3x &= 16 \\ x &= \frac{16}{3} \\ x &= 5\frac{1}{3} \end{aligned}$$

7.
$$\begin{aligned} 5x - 3yx + y^2 &= 5(2) - 3(1)(2) + (1)^2 \\ &= 10 - 6 + 1 \\ &= 5 \end{aligned}$$

8. $P(3)=5(3)^2-3(3)+7$

$=5\times9-9+7$

$=43$

9. $\left(2x^2-3x+7\right)+\left(x^2-4x-8\right)$

$=\left(2x^2+x^2\right)+(-3x-4x)+(7-8)$

$=3x^2-7x-1$

10. $\left(2x^2-5x+8\right)-\left(x^2-3x+2\right)$

$=2x^2-5x+8-x^2+3x-2$

$=\left(2x^2-x^2\right)+(-5x+3x)+(8-2)$

$=x^2-2x+6$

11. $\left(4x^2y\right)\left(21xy^3\right)$

$=(4\times21)\left(x^2\times x\right)\left(y\times y^3\right)$

$=84x^3y^4$

12. $3\left(2x^2-5y\right)+2x(3x+y)$

$=6x^2-15y+6x^2+2xy$

$=12x^2+2xy-15y$

13. $(2x+1)(x-3)$

$=(2x)(x)+(2x)(-3)+(1)(x)+(1)(-3)$

$=2x^2-6x+x-3$

$=2x^2-5x-3$

14. $(2x-3)\left(x^2-x+2\right)$

$=(2x)\left(x^2\right)+(2x)(-x)+(2x)(2)+(-3)\left(x^2\right)+(-3)(-x)+(-3)(2)$

$=2x^3-2x^2+4x-3x^2+3x-6$

$=2x^3-5x^2+7x-6$

15. $2x^2+4x-30$

$=2\left(x^2+2x-15\right)$

$=2(x+5)(x-3)$

16. Step 1: Let x be the length of the table. 3 times the length of the table = $3x$.

Step 2: $3x+9=36$

Step 3: $3x=(36-9)$

$3x=27$

$x=\frac{27}{3}$

$x=9$

Step 4: Does 3 times the length of the table plus 9 cm equal 36 cm?

$$[3(9) + 9] = (27 + 9)$$
$$= 36$$

Therefore, the length of the table is 9 cm.

If you had trouble with the **Review**, go to Grade 9 mathematics and/or Mathematics 13, Units 1 and 2.

Exploring Topic 1

Activity 1

Translate English sentences into Algebra.

1. a. $x + 3$ b. $x - 7$
   c. $\frac{8}{x}$ d. $6x$
   e. $5 - x$

2. a. x = my age $2x$ = Jim's age
   b. x = Joe's earnings $x + 300$ = Jean's earnings
   c. x cm = width $(x + 5)$ cm = length
   d. x = number of girls $2x - 4$ = number of boys

Activity 2

Solve and verify simple linear equations with integral coefficients.

1. a.

$$\begin{aligned} 3x-2(x+4) &= 5-3(x-1) \\ 3x-2x-8 &= 5-3x+3 \\ x-8 &= 8-3x \\ x+3x &= 8+8 \\ 4x &= 16 \\ x &= 4 \end{aligned}$$

b.

$$\begin{aligned} 2+3(x-5) &= x-2(x+3) \\ 2+3x-15 &= x-2x-6 \\ 3x-13 &= -x-6 \\ 3x+x &= 13-6 \\ 4x &= 7 \\ x &= \frac{7}{4} \end{aligned}$$

2. Let x = smaller number.
   Then $2x + 8$ = larger number.

$$\begin{aligned} (2x+8)-x &= 15 \\ 2x+8-x &= 15 \\ x+8 &= 15 \\ x &= 7 \qquad 2x+8 = 2\times7+8 \\ &\qquad\qquad\quad = 22 \end{aligned}$$

The two numbers are 7 and 22.

3. Let x = measure of ∠B.
Then $x + 5$ = measure of ∠A and
$x - 35$ = measure of ∠C.

$$\begin{aligned}(x)+(x+5)+(x-35)&=180\\3x-30&=180\\3x&=210\\x&=70\\x+5&=75\\x-35&=35\end{aligned}$$

The measures of the angles are ∠A = 75°, ∠B = 70° and ∠C = 35°.

4. Let x = son's age.
Then $3x + 8$ = mother's age.
Four years ago: Son's age = $x - 4$
Mother's age = $3x + 8 - 4 = 3x + 4$ or $11(x - 4)$

$$\begin{aligned}3x+4&=11(x-4)\\3x+4&=11x-44\\44+4&=11x-3x\\48&=8x\\x&=\frac{48}{8}\\x&=6\\3x+8&=3\times6+8\\&=26\end{aligned}$$

The mother's age is 26 years.

5. Let x = width.
$x + 5$ = length.

$$\begin{aligned}2(x)+2(x+5)&=90\\2x+2x+10&=90\\4x&=80\\x&=20\\x+5&=25\end{aligned}$$

The dimensions of the rectangle is 20 cm by 25 cm.

6. Let x = number of nickels.
$x + 5$ = number of dimes.
$x - 2$ = number of quarters.

$$\begin{aligned}5x+10(x+5)+25(x-2)&=200\\5x+10x+50+25x-50&=200\\40x&=200\\x&=5\text{ (nickels)}\\x+5&=5+5\\&=10\text{ (dimes)}\\x-2&=5-2\\&=3\text{ (quarters)}\end{aligned}$$

There are 5 nickels, 10 dimes, and 3 quarters.

Activity 3

Solve and verify simple linear equations with rational coefficients.

1. Step 1: Let t = time it takes for this car to travel a distance of 300 km.

   Step 2: Since distance = speed × time,

   $$300 = 1\frac{2}{3} \times t.$$

   Step 3: Solve $300 = 1\frac{2}{3}t$.

   $$\frac{5}{3}t = 300$$

   $$t = 300 \times \frac{3}{5}$$

   $$= 180$$

   Step 4: $1\frac{2}{3} \times 180 = 300$

   It takes 180 minutes to travel 300 km.

2. Step 1: Let x = time it takes for John to walk to school in minutes.

   Step 2: Distance = speed × time

   $$1\text{km} = 1000 \text{ m}$$

   $$1000 = 66\frac{2}{3} \times x$$

   Step 3: $1000 = \frac{200}{3} \times x$

   $$1000 \times \frac{3}{200} = x$$

   $$\frac{3000}{200} = x$$

   $$15 = x$$

   Step 4: Substitute 15 min into $1000 = \frac{200}{3}x$.

LS	RS
1000	$\frac{200}{3}(15)$
	1000

   LS = RS

   It takes John 15 minutes to walk to school.

3. Step 1: Let x = smaller number.
   $(x + 1)$ = larger number.

   Step 2: $\frac{1}{2}(x) + \frac{1}{3}(x+1) = \frac{7}{6}$

   Step 3: Multiply every term by 6.

   $$\overset{3}{\cancel{6}} \times \frac{1}{\underset{1}{\cancel{2}}}(x) + \overset{2}{\cancel{6}} \times \frac{1}{\underset{1}{\cancel{3}}}(x+1) = \frac{7}{\underset{1}{\cancel{6}}} \times \overset{1}{\cancel{6}}$$

   $$3x + 2(x+1) = 7$$

   $$3x + 2x + 2 = 7$$

   $$5x + 2 = 7$$

   $$5x = 5$$

   $$x = 1$$

   $$x + 1 = 2$$

Step 4: $LS=\frac{1}{2}(1)+\frac{1}{3}(2)$

$$=\frac{1}{2}+\frac{2}{3}$$

$$=\frac{7}{6}$$

$$=RS$$

The two numbers are 1 and 2.

4. Step 1: Let x = smaller number.
$x + 1$ = larger number.

Step 2: $\frac{1}{3}x-\frac{1}{4}(x+1)=\frac{1}{12}$

$$\overset{4}{\cancel{12}}\times\frac{1}{\underset{1}{\cancel{3}}}x-\overset{3}{\cancel{12}}\times\frac{1}{\underset{1}{\cancel{4}}}(x+1)=\overset{1}{\cancel{12}}\times\frac{1}{\underset{1}{\cancel{12}}}$$

Step 3: $4x-3x-3=1$

$$x-3=1$$

$$x=4$$

$$x+1=5$$

The two numbers are 4 and 5.

5. $\frac{2}{3}x=8$

Common denominator = 3

$$\left(\overset{1}{\cancel{3}}\right)\left(\frac{2}{\underset{1}{\cancel{3}}}x\right)=(3)\times(8)$$

$$2x=24$$

$$x=12$$

Verify x = 12.

LS	RS
$\frac{2}{3}x$	8
$\frac{2}{3}(12)$	
8	

LS = RS

6. $\frac{a}{5}=-3$

Common denominator = 5

$$\left(\overset{1}{\cancel{5}}\right)\left(\frac{a}{\underset{1}{\cancel{5}}}\right)=(5)(-3)$$

$$a=-15$$

Verify a = – 15.

LS	RS
$\frac{a}{5}$	– 3
$\frac{-15}{5}$	
-3	

LS = RS

7. $\frac{x}{5}-\frac{x}{6}=10$

Common denominator = 30

$$\overset{6}{\cancel{30}}\left(\frac{x}{\underset{1}{\cancel{5}}}\right)-\overset{5}{\cancel{30}}\left(\frac{x}{\underset{1}{\cancel{6}}}\right)=(30)(10)$$

$$6x-5x=300$$

$$x=300$$

Verify x = 300.

LS	RS
$\frac{x}{5}-\frac{x}{6}$	10
$\frac{300}{5}-\frac{300}{6}$	
$60-50$	
10	

LS = RS

8. $\frac{x}{3}+\frac{x}{5}=4$

Common denominator = 15

$$\overset{5}{\cancel{15}}\left(\frac{x}{\underset{1}{\cancel{3}}}\right)+\overset{3}{\cancel{15}}\left(\frac{x}{\underset{1}{\cancel{5}}}\right)=(15)(4)$$

$$5x+3x=60$$

$$8x=60$$

$$x=7.5$$

Verify $x=7.5$.

LS	RS
$\frac{x}{3}+\frac{x}{5}$	4
$\frac{7.5}{3}+\frac{7.5}{5}$	
$2.5+1.5$	
4	

LS = RS

9. $\frac{x-1}{3}=\frac{x+2}{4}$

Common denominator = 12

$$\overset{4}{\cancel{12}}\left(\frac{x-1}{\underset{1}{\cancel{3}}}\right)=\overset{3}{\cancel{12}}\left(\frac{x+2}{\underset{1}{\cancel{4}}}\right)$$

$$4(x-1)=3(x+2)$$

$$4x-4=3x+6$$

$$4x-3x=6+4$$

$$x=10$$

Verify $x = 10$.

LS	RS
$\frac{x-1}{3}$	$\frac{x+2}{4}$
$\frac{10-1}{3}$	$\frac{10+2}{4}$
$\frac{9}{3}$	$\frac{12}{4}$
3	3

LS = RS

10. $\frac{2x-1}{3}=\frac{x}{5}$

Common denominator = 15

$$\left(\overset{5}{\cancel{15}}\right)\frac{2x-1}{\underset{1}{\cancel{3}}}=\left(\overset{3}{\cancel{15}}\right)\frac{x}{\underset{1}{\cancel{5}}}$$

$$5(2x-1)=3x$$

$$10x-5=3x$$

$$10x-3x=5$$

$$7x=5$$

$$x=\frac{5}{7}$$

Verify $x=\frac{5}{7}$.

LS	RS
$\frac{2x-1}{3}$	
$\frac{2\left(\frac{5}{7}\right)-1}{3}$	$\frac{\frac{5}{7}}{5}$
$\frac{\frac{10}{7}-\frac{7}{7}}{3}$	$\frac{5}{7}\times\frac{1}{5}$
$\frac{3}{7}\times\frac{1}{3}$	$\frac{1}{7}$
$\frac{1}{7}$	

LS = RS

11. $\frac{x+2}{3}-\frac{x-3}{4}=\frac{3}{2}$

Common denominator = 12

$$\left(\overset{4}{\cancel{12}}\right)\left(\frac{x+2}{\underset{1}{\cancel{3}}}\right)-\left(\overset{3}{\cancel{12}}\right)\left(\frac{x-3}{\underset{1}{\cancel{4}}}\right)=\left(\overset{6}{\cancel{12}}\right)\left(\frac{3}{\cancel{2}}\right)$$

$$4(x+2)-3(x-3)=6\times 3$$

$$4x+8-3x+9=18$$

$$x+17=18$$

$$x=1$$

Verify $x=1$.

LS	RS
$\frac{x+2}{3}-\frac{x-3}{4}$	$\frac{3}{2}$
$\frac{1+2}{3}-\frac{1-3}{4}$	
$\frac{3}{3}-\frac{-2}{4}$	
$1+\frac{1}{2}$	
$\frac{3}{2}$	

LS = RS

12. $\frac{x+5}{3}-\frac{x-1}{2}=\frac{1}{6}$

Common denominator = 6

$$\left(\overset{2}{\cancel{6}}\right)\left(\frac{x+5}{\underset{1}{\cancel{3}}}\right)-\left(\overset{3}{\cancel{6}}\right)\left(\frac{x-1}{\underset{1}{\cancel{2}}}\right)=\left(\overset{1}{\cancel{6}}\right)\left(\frac{1}{\underset{1}{\cancel{6}}}\right)$$

$$2(x+5)-3(x-1)=1$$

$$2x+10-3x+3=1$$

$$-x+13=1$$

$$-x=-12$$

$$x=12$$

Verify $x=12$.

LS	RS
$\frac{x+5}{3}-\frac{x-1}{2}$	$\frac{1}{6}$
$\frac{12+5}{3}-\frac{12-1}{2}$	
$\frac{17}{3}-\frac{11}{2}$	
$\frac{34}{6}-\frac{33}{6}$	
$\frac{1}{6}$	

LS = RS

Extra Help

1. $\frac{4x}{(4)}=\frac{36}{(4)}$

$$x=(9)$$

Verify $x=9$.

LS	RS
$4x$	36
$4(9)$	
36	

LS = RS

2. $3y-1+(1)=5+(1)$

$$3y=(6)$$

$$\frac{3y}{(3)}=\frac{(6)}{(3)}$$

$$y=(2)$$

Verify $y=2$.

LS	RS
$3y-1$	5
$3(2)-1$	
5	

LS = RS

3. $2x+2+(3)x-(21)=6$

$$(5)x-(19)=6$$

$$(5)x=(25)$$

$$x=\left(\frac{25}{5}\right)$$

$$x=(5)$$

Verify $x=5$.

LS	RS
$2(x+1)+3(x-7)$	6
$2(5+1)+3(5-7)$	
$2(6)+3(-2)$	
6	

LS = RS

4. $3x+(\mathbf{3})-2x+2=7$

The 4th term is positive because (–2) (–1) = 2.

$$(\mathbf{1})x+(\mathbf{5})=7$$
$$x=(\mathbf{2})$$

Verify $x = 2$.

LS	RS
$3(x+1)-2(x-1)$	7
$3(2+1)-2(2-1)$	
$3(3)-2(1)$	
$9-2$	
7	

LS = RS

5. $\frac{3x}{\underset{1}{\cancel{4}}}\left(\overset{3}{\cancel{12}}\right)-\frac{5x}{\underset{1}{\cancel{6}}}\left(\overset{2}{\cancel{12}}\right)=\frac{1}{\underset{1}{\cancel{3}}}\left(\overset{4}{\cancel{12}}\right)$

$$(\mathbf{9})x-(\mathbf{10})x=(\mathbf{4})$$
$$(\mathbf{-})x=(\mathbf{4})$$
$$x=(\mathbf{-4})$$

Verify $x = -4$.

LS	RS
$\frac{3x}{4}-\frac{5x}{6}$	$\frac{1}{3}$
$\frac{3(-4)}{4}-\frac{5(-4)}{6}$	
$-3+\frac{-20}{6}$	
$\frac{-9}{3}+\frac{10}{3}$	
$\frac{1}{3}$	

LS = RS

6. a. $3x = 81$
$$x=\frac{81}{3}$$
$$x = 27$$

b. $7x = 56$
$$x=\frac{56}{7}$$
$$x = 8$$

7. a. $5x-3x+8=x+11$
$$2x+8=x+11$$
$$2x+8-8=x+11-8$$
$$2x=x+3$$
$$2x-x=x+3-x$$
$$x=3$$

b. $7x+x-3=2x+13$
$$8x-3=2x+13$$
$$8x-3+3=2x+13+3$$
$$8x=2x+16$$
$$8x-2x=2x+16-2x$$
$$6x=16$$
$$\frac{6x}{6}=\frac{16}{6}$$
$$x=2\frac{2}{3}$$

8. a. $\frac{x}{4}=13$
$$x=4\times 13$$
$$x=52$$

b. $\frac{-x}{3}=9$
$$\left(\frac{\overset{-1}{\cancel{-3}}}{}\right)\left(\frac{-x}{\underset{1}{\cancel{3}}}\right)=(-3)9$$
$$x=-27$$

9. a. $\frac{x}{4}+3=13$

$$\left(\overset{1}{\cancel{4}}\right)\left(\frac{x}{\underset{1}{\cancel{4}}}\right)+(4)(3)=(4)(13)$$

$$x+12=52$$

$$x=40$$

b. $\frac{x}{5}-4=-1$

$$\left(\overset{1}{\cancel{5}}\right)\left(\frac{x}{\underset{1}{\cancel{5}}}\right)-(5)(4)=(5)(-1)$$

$$x-20=-5$$

$$x-20+20=-5+20$$

$$x=15$$

10. a. $\frac{x}{5}+\frac{1}{2}=\frac{1}{3}$

Multiply every term by 30.

$$\left(\overset{6}{\cancel{30}}\right)\left(\frac{x}{\underset{1}{\cancel{5}}}\right)+\left(\overset{15}{\cancel{30}}\right)\left(\frac{1}{\underset{1}{\cancel{2}}}\right)=\overset{10}{\cancel{30}}\left(\frac{1}{\underset{1}{\cancel{3}}}\right)$$

$$6x+15=10$$

$$6x+15-15=10-15$$

$$6x=-5$$

$$x=\frac{-5}{6}$$

b. $\frac{x}{2}-\frac{x}{4}=3$

Multiply every term by 8.

$$\left(\overset{4}{\cancel{8}}\right)\left(\frac{x}{\underset{1}{\cancel{2}}}\right)-\overset{2}{\cancel{8}}\left(\frac{x}{\underset{1}{\cancel{4}}}\right)=8(3)$$

$$4x-2x=24$$

$$2x=24$$

$$x=12$$

Extensions

1. Let x = smaller even number.
   $(x+2)$ = larger even number.

$$\frac{1}{3}x-\frac{1}{4}(x+2)=\frac{1}{3} \qquad \text{(L.C.D. is 12.)}$$

$$\left(\overset{4}{\cancel{12}}\right)\left(\frac{1}{\underset{1}{\cancel{3}}}x\right)-\left(\overset{3}{\cancel{12}}\right)\left(\frac{1}{\underset{1}{\cancel{4}}}\right)(x+2)=\left(\overset{4}{\cancel{12}}\right)\left(\frac{1}{\underset{1}{\cancel{3}}}\right)$$

$$4x-3x-6=4$$

$$x-6=4$$

$$x=10$$

$$x+2=12$$

The two numbers are 10 and 12.

2. Let x = 1st number.
   $(x+1)$ = 2nd number.
   $(x+2)$ = 3rd number.

$$\frac{1}{3}x+\frac{1}{5}(x+1)+\frac{1}{2}(x+2)=26 \qquad \text{(L.C.D.} = 30.)$$

$$\overset{10}{\cancel{30}}\times\frac{1}{\underset{1}{\cancel{3}}}x+\overset{6}{\cancel{30}}\times\frac{1}{\underset{1}{\cancel{5}}}(x+1)+\overset{15}{\cancel{30}}\times\frac{1}{\underset{1}{\cancel{2}}}(x+2)=26\times30$$

$$10x+6x+6+15x+30=780$$

$$31x+36=780$$

$$31x=744$$

$$x=24$$

Then $x+1=25$ and $x+2=26$.

The three consecutive numbers are 24, 25, and 26.

3. Let x = Sara's monthly income.

Sara's monthly contribution = $\frac{x}{10}$.

Jack's monthly contribution = $\frac{x}{2}-2$.

Ron's monthly contribution = $\frac{3}{4}x$.

$$\frac{x}{10}+\left(\frac{x}{2}-2\right)+\frac{3}{4}x=4048$$

Multiply every term by 20.

$$\overset{2}{\cancel{20}}\left(\frac{x}{\underset{1}{\cancel{10}}}\right)+(20)\left(\frac{x}{2}-2\right)+\left(\overset{5}{\cancel{20}}\right)\left(\frac{3}{\underset{1}{\cancel{4}}}x\right)=20\times4048$$

$$2x+10x-40+15x=80\,960$$

$$27x=81\,000$$

$$x=\frac{81\,000}{27}$$

$$x=\$3000$$

Sara's monthly income is \$3000.

Exploring Topic 2

Activity 1

Apply the Reverse the Sign Rule to solve and graph the solution of a linear inequality.

1. a. $x-5\geq 11$

$$x\geq 11+5$$

$$x\geq 16$$

Verify $x = 12$.

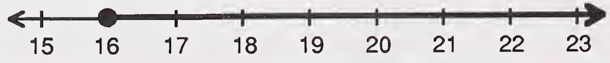
LS	RS
$12-5$	11
7	

LS $\ngeq$ RS

Verify $x = 20$.

LS	RS
$20-5$	11
15	

LS $\geq$ RS



b. $-3+5x<12$

$5x<15$

$x<3$

Verify $x=2$.

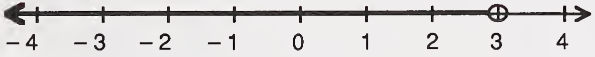
LS	RS
$-3+5(2)$	12
$-3+10$	
7	

LS < RS

Verify $x=5$.

LS	RS
$-3+5(5)$	12
$-3+25$	
22	

LS ≮ RS



c. $3y-2>y+6$

$3y-y>6+2$

$2y>8$

$y>4$

Verify $y=6$.

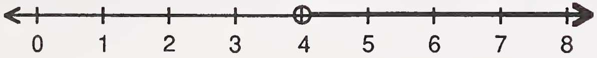
LS	RS
$3(6)-2$	$6+6$
$18-2$	12
16	

LS > RS

Verify $y=2$.

LS	RS
$3(2)-2$	$2+6$
$6-2$	8
4	

LS ≯ RS



d. $y+3<2y-7$

$y<2y-10$

$y-2y<2y-10-2y$

$-y<-10$

$y>10$

Divide both sides by (–1) and reverse the inequality sign.

Verify $y=9$.

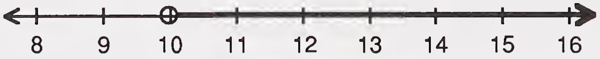
LS	RS
$9+3$	$2(9)-7$
12	$18-7$
	11

LS ≮ RS

Verify $y=11$.

LS	RS
$11+3$	$2(11)-7$
14	$22-7$
	15

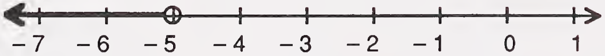
LS < RS



2. a. $3a<-15$

$\frac{3a}{3}<\frac{-15}{3}$

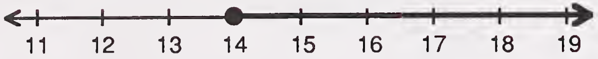
$a<-5$



b. $\frac{x}{7} \ge 2$

$\frac{x}{\underset{1}{\cancel{7}}}\left(\overset{1}{\cancel{7}}\right) \ge (2)(7)$

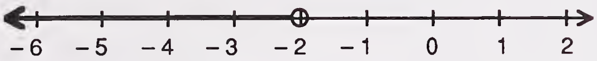
$x \ge 14$



c. $-3y > 6$

$\frac{\left(\overset{1}{\cancel{-3}}y\right)}{\left(\underset{1}{\cancel{-3}}\right)} < \frac{\overset{2}{\cancel{6}}}{\left(\underset{1}{\cancel{-3}}\right)}$

$y < -2$



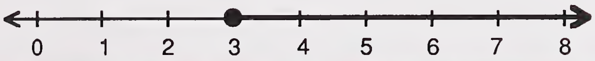
d. $\frac{-2x}{3} \le -2$

$\frac{-2x}{\underset{1}{\cancel{3}}}\left(\overset{1}{\cancel{3}}\right) \le (-2)(3)$

$-2x \le -6$

$\frac{\overset{1}{\cancel{-2}}x}{\underset{1}{\cancel{-2}}} \ge \frac{\left(\overset{3}{\cancel{-6}}\right)}{\left(\underset{1}{\cancel{-2}}\right)}$

$x \ge 3$



3. a. $x > 3$

b. $x \le -1$

c. $x > -2$

d. $x \ge 2$

4. a. $2 - 3x \ge -19$

$-3x \ge -21$

$\frac{\overset{1}{\cancel{-3}}x}{\left(\underset{1}{\cancel{-3}}\right)} \le \frac{\overset{7}{\cancel{-21}}}{\left(\underset{1}{\cancel{-3}}\right)}$

$x \le 7$

b. $-\frac{1}{3}x + 3 < -9$

$-\frac{1}{3}x < -12$

$\left(\frac{-1}{\underset{1}{\cancel{3}}}x\right)\left(\overset{1}{\cancel{-3}}\right) > (-12)(-3)$

$x > 36$

c. $-3(2x-4) > 4(x-1)-14$

$-6x+12 > 4x-4-14$

$-6x+12 > 4x-18$

$-6x-4x+12 > 4x-18-4x$

$-10x+12 > -18$

$$\frac{\overset{1}{\cancel{-10}}x}{\left(\underset{1}{\cancel{-10}}\right)} < \frac{\overset{3}{\cancel{-30}}}{\left(\underset{1}{\cancel{-10}}\right)}$$

$x < 3$

d. $5(x-3) < -2(x+2)+11$

$5x-15 < -2x-4+11$

$5x-15 < -2x+7$

$5x+2x < 7+15$

$7x < 22$

$x < \frac{22}{7}$

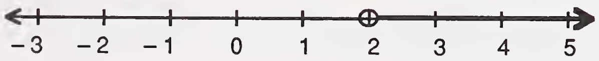
Extra Help

1. Subtract 5 from both sides.

$x+5-(5) > 7-(5)$

$x > (2)$

Graph:



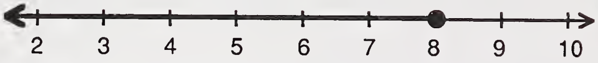
Open dot, because 2 is not included.

2. Add 5 to both sides.

$x-5+(5) \le 3+(5)$

$x \le (8)$

Graph:



Solid dot, because 8 is included (≤).

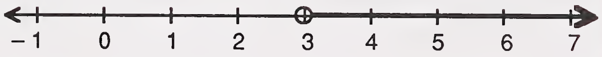
3. Divide both sides by 5.

$$\frac{5x}{(5)} > \frac{15}{(5)}$$

$x > (3)$

No. You don't reverse the inequality sign if you divide both sides by a positive number.

Graph:



4. Divide both sides by (– 5) and change the direction of the inequality.

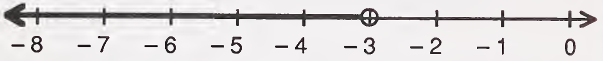
$-5x > 15$

$$\frac{-5x}{(-5)} < \frac{15}{(-5)}$$

$x < (-3)$

Yes. You have to reverse the inequality sign if you divide each side by a negative number.

Graph:



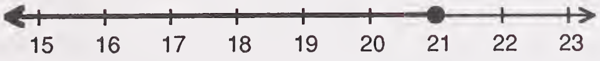
5. Multiply both sides by 3.

$\frac{x}{3}(3) \le 7(3)$

$x \le (21)$

No. You don't reverse the inequality sign if you multiply both sides by a positive number.

Graph:



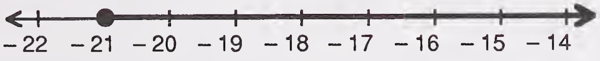
6. Multiply both sides by (– 3).

$\frac{x}{-3}(-3) \ge 7(-3)$

$x \ge (-21)$

Yes. You have to reverse the inequality sign if you multiply both sides by a negative number.

Graph:

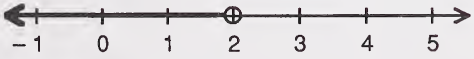


7. $x + 3 < 5$

$x < 5 - 3$

$x < 2$

Graph:

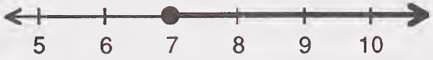


8. $x - 4 \ge 3$

$x - 4 + 4 \ge 3 + 4$

$x \ge 7$

Graph:

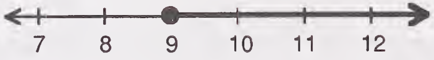


9. $3x \ge 27$

$\frac{3x}{3} \ge \frac{27}{3}$

$x \ge 9$

Graph:

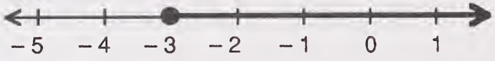


10. $-4x \le 12$

$\frac{-4x}{(-4)} \ge \frac{12}{(-4)}$

$x \ge -3$

Graph:

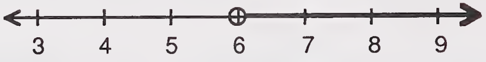


11. $\frac{x}{4} > \frac{3}{2}$

$\frac{x}{4}(4) > \frac{3}{2}(4)$

$x > 6$

Graph:

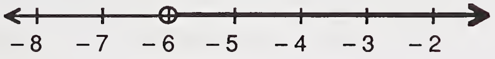


12. $\frac{-x}{3} < 2$

$\frac{-x}{3}(-3) > 2(-3)$

$x > -6$

Graph:



Extensions

1. $\frac{x}{2} - \frac{2}{7}x + \frac{2}{3} \le \frac{1}{3}$

Multiply every term by 42, the L.C.D.

$21x - 12x + 28 \le 14$

$9x + 28 \le 14$

$9x \le -14$

$x \le -\frac{14}{9}$

2. $x + \frac{1}{5}x - \frac{1}{2} > \frac{1}{10}$

Multiply every term by 10.

$10x + 2x - 5 > 1$

$12x - 5 > 1$

$12x > 6$

$x > \frac{6}{12}$

$x > \frac{1}{2}$

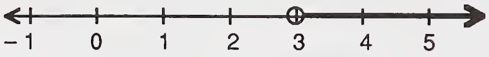
3. $x + 3(x+1) > 15$

$x + 3x + 3 > 15$

$4x + 3 > 15$

$4x > 12$

$x > 3$

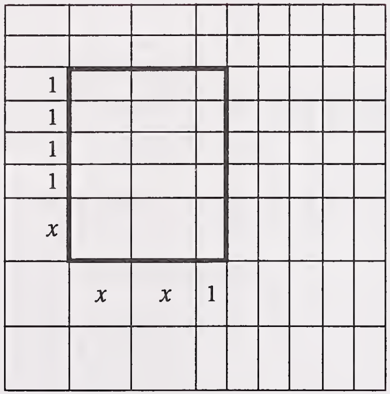


Exploring Topic 3

Activity 1

Factor a trinomial of the form $ax^2 + bx + c$, where a, b, and c are integers.

1. $2x^2+9x+4$



$2x^2+9x+4=(2x+1)(x+4)$

Algebraic method: $2x^2+9x+4$
Factors of first term are $2x$ and $1x$. Therefore, possible first terms of the binomial factors are $(2x\quad)\ (x\quad)$. Factors of the constant term are (1)(4), (2)(2). Thus possible factors are
$(2x+1)(x+4)$

$(2x+4)(x+1)$

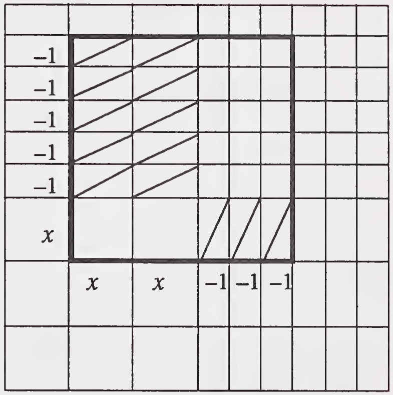
$(2x+2)(x+2)$

Check the middle term of the possible factors.

For $(2x+1)(x+4)$, the middle term is $1x + 8x = 9x$.

This is correct. Thus, $2x^2+9x+4=(2x+1)(x+4)$

2. $2x^2-13x+15$



$2x^2-13x+15=(x-5)(2x-3)$

Algebraic Method: $2x^2-13x+15$
The factors of the first term are $(1x)\ (2x)$. Therefore, possible 1st terms of the factors are $(1x\quad)\ (2x\quad)$. Factors of the constant term are (–1) (–15) and (–5) (–3). Both must be negative because the middle term is negative and the constant term is positive. Thus, the possible factors are

$(x-15)\ (2x-1)$
$(x-1)\ (2x-15)$
$(x-5)\ (2x-3)$
$(x-3)\ (2x-5)$

Now check the middle term of the possible factors.

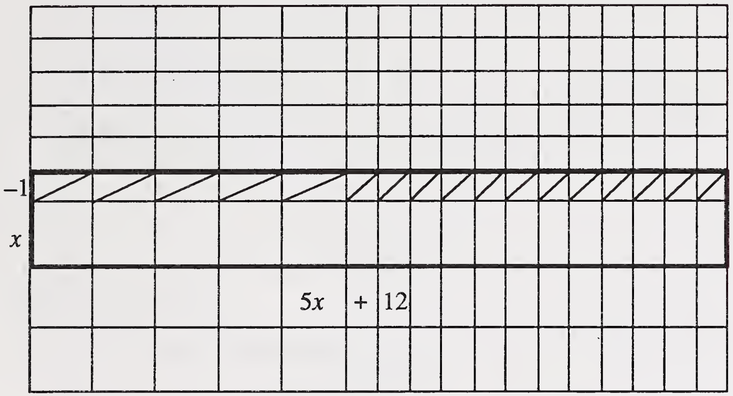
For $(x-15)(2x-1)$, the middle term is $-30x-x=-31x$.

For $(x-1)(2x-15)$, the middle term is $-2x-15x=-17x$.

For $(x-5)(2x-3)$, the middle term is $-10x-3x=-13x$.

This one is correct.
Therefore, $2x^2-13x+15=(x-5)(2x-3)$.

3. $5x^2+7x-12$



$$5x^2+7x-12=(x-1)(5x+12)$$

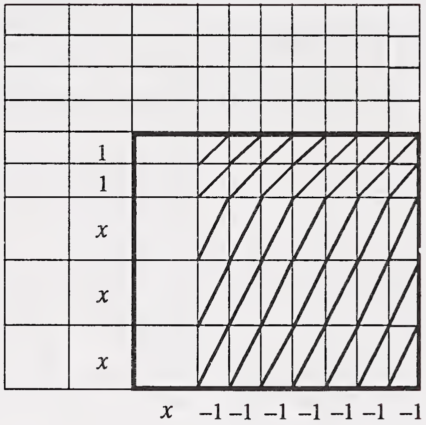
Algebraic Method: $5x^2+7x-12$
The factors of the first term are $(1x)(5x)$. Therefore, possible first terms of the binomial factors are $(1x\quad)(5x\quad)$. Since the constant term is negative, one of the factors of 12 must be positive and one negative. Therefore, factors of the constant term are (1)(–12), (+12)(–1), (2)(– 6), (+6)(– 2), (3)(– 4), or (+ 4)(– 3). Thus, possible factors are

$(x + 1)(5x - 12)$
$(x - 12)(5x + 1)$
$(x + 12)(5x - 1)$
$(x - 1)(5x + 12)$
.
.
.
$(x - 4)(5x + 3)$

Check the middle terms of the possible factors.
For $(x + 1)(5x - 12)$, the middle term is $5x - 12x = -7x$.
For $(x - 12)(5x + 1)$, the middle term is $-60x + x = -59x$.
For $(x + 12)(5x - 1)$, the middle term is $60x - x = 59x$.
For $(x - 1)(5x + 12)$, the middle term is $-5x + 12x = +7x$.

This one is correct.
Therefore, $5x^2+7x-12=(x-1)(5x+12)$.

4. $3x^2-19x-14$



$$3x^2-19x-14=(x-7)(3x+2)$$

Algebraic Method: $3x^2-19x-14=(x-7)(3x+2)$
The factors of the first term are $(1x)(3x)$. Therefore, possible first terms of the binomial factors are $(x\quad)(3x\quad)$. Since the constant term is negative, one of the factors of 14 must be negative. Therefore, factors of the constant term are (–1) (14), (1) (–14), (2) (–7) or (–2) (7).

Thus, possible factors are

$(x-1)(3x+14)$
$(x+1)(3x-14)$
$(x+14)(3x-1)$
$(x-14)(3x+1)$
$(x+2)(3x-7)$
$(x-2)(3x+7)$
$(x-7)(3x+2)$
$(x+7)(3x-2)$

Check the middle terms of these possible factors.

For $(x-1)(3x+14)$, the middle term is $-3x+14x=11x$.

For $(x+1)(3x-14)$, the middle term is $+3x-14x=-11x$.

For $(x+14)(3x-1)$, the middle term is $42x-x=+41x$.

For $(x-14)(3x+1)$, the middle term is $-42x+x=-41x$

For $(x+2)(3x-7)$, the middle term is $6x-7x=-x$.

For $(x-2)(3x+7)$, the middle term is $-6x+7x=+x$.

For $(x-7)(3x+2)$, the middle term is $-21x+2x=-19x$.

This one is correct.
Therefore, $3x^2-19x-14=(x-7)(3x+2)$.

5. $6x^2+5x+1$

Factors of the first term are $(1x)(6x)$ and $(2x)(3x)$. Therefore, possible 1st terms of the binomial factors are $(x\quad)(6x\quad)$ and $(2x\quad)(3x\quad)$. Factors of the constant term are (1)(1). Thus, possible factors of the trinomial are

$(x+1)(6x+1)$
$(2x+1)(3x+1)$

Check the middle terms of these possible factors.

For $(x+1)(6x+1)$, the middle term is $6x+1x=7x$.

For $(2x+1)(3x+1)$, the middle term is $3x+2x=5x$.

This is correct.
Therefore, $6x^2+5x+1=(2x+1)(3x+1)$.

6. $6x^2-11x+3$

Factors of the first term are $(1x)(6x)$ and $(2x)(3x)$. Therefore, possible first terms of the binomial factors are $(x\quad)(6x\quad)$ and $(2x\quad)(3x\quad)$. Factors of the constant term are $(-1)(-3)$. The factors are negative because the middle term is negative and the constant is positive. Thus, possible factors are

$(x-1)(6x-3)$
$(x-3)(6x-1)$
$(2x-3)(3x-1)$
$(2x-1)(3x-3)$

Check the middle terms of these possible factors.

For $(x-1)(6x-3)$, the middle term is $-6x-3x=-9x$.

For $(x-3)(6x-1)$, the middle term is $-18x-x=-19x$.

For $(2x-3)(3x-1)$, the middle term is $-9x-2x=-11x$.

This one is correct.
Therefore, $6x^2-11x+3=(2x-3)(3x-1)$.

7. $3x^2+7x-6$

The factors of the first term are $(1x)(3x)$. Therefore, possible first terms of the binomial factors are $(1x \quad)(3x \quad)$. Since the constant term is negative, one of the factors of 6 must be negative and one positive. Therefore, factors of the constant term are $(1)(-6)$, $(-1)(6)$, $(-2)(3)$ and $(2)(-3)$. Thus, possible factors are

$(x+1)(3x-6)$
$(x-6)(3x+1)$
$(x-1)(3x+6)$
$(x+6)(3x-1)$
$(x-2)(3x+3)$
$(x+2)(3x-3)$
$(x+3)(3x-2)$
$(x-3)(3x+2)$

Check the middle terms of the possible factors.

For $(x+1)(3x-6)$, the middle term is $3x-6x=-3x$.

For $(x-6)(3x+1)$, the middle term is $-18x+1x=-17x$.

For $(x-1)(3x+6)$, the middle term is $-3x+6x=3x$.

For $(x+6)(3x-1)$, the middle term is $18x-x=+17x$.

For $(x-2)(3x+3)$, the middle term is $-6x+3x=-3x$.

For $(x+3)(3x-2)$, the middle term is $9x-2x=7x$.

This one is correct.
Therefore, $3x^2+7x-6=(x+3)(3x-2)$.

8. $15x^2-x-2$

The factors of the first term are $(x)(15x)$ and $(3x)(5x)$. Therefore, possible first terms are $(x \quad)(15x \quad)$ and $(3x \quad)(5x \quad)$. Since the constant term is negative, one of the factors of -2 must be negative and one positive. Therefore, the factors of the constant term are $(-1)(2)$ and $(1)(-2)$. Thus, the possible factors are

$(x-1)(15x+2)$
$(x+1)(15x-2)$
$(x+2)(15x-1)$
$(x-2)(15x+1)$
$(3x-1)(5x+2)$
$(3x+1)(5x-2)$
$(3x+2)(5x-1)$
$(3x-2)(5x+1)$

Check the middle terms of these possible factors.

For $(x-1)(15x+2)$, the middle term is $-15x+2x=-13x$.

For $(x+1)(15x-2)$, the middle term is $+15x-2x=+13x$.

For $(x+2)(15x-1)$, the middle term is $30x-x=29x$.

For $(x-2)(15x+1)$, the middle term is $-30x+x=-29x$.

For $(3x-1)(5x+2)$, the middle term is $-5x+6x=+x$.

For $(3x+1)(5x-2)$, the middle term is $+5x-6x=-x$.

This one is correct.
Therefore, $15x^2-x-2=(3x+1)(5x-2)$.

Activity 2

Factor a perfect trinomial square.

1. $16x^2+24x+9$
$=(4x)^2+2(4)(3)x+3^2$
$=(4x+3)^2$

2. $25x^2+70x+49$
$=(5x)^2+2(5)(7)x+(7)^2$
$=(5x+7)^2$

3. $36x^2-60x+25$
$=(6x)^2-(2)(6)(5)x+(5)^2$
$=(6x-5)^2$

4. $9x^2-6x+1$
$=(3x)^2-2(3)(1)(x)+1^2$
$=(3x-1)^2$

Activity 3

Factor trinomials by applying more than one factoring method.

1. a. $5x^2+35x+50$
$=5(x^2+7x+10)$
$=5(x+2)(x+5)$

b. $2x^2-2x-112$
$=2(x^2-x-56)$
$=2(x-8)(x+7)$

2. a. $3y^3-15y^2-42y$
$=3y(y^2-5y-14)$
$=3y(y-7)(y+2)$

b. $5y^3-60y^2+180y$
$=5y(y^2-12y+36)$
$=5y(y-6)^2$

3. a. $(4a^2+12a+9)-(b^2-10b+25)$
$=(2a+3)^2-(b-5)^2$
$=[(2a+3)+(b-5)][(2a+3)-(b-5)]$
$=(2a+3+b-5)(2a+3-b+5)$
$=(2a+b-2)(2a-b+8)$

b. $(4x^2-8x+4)-(y^2+4y+4)$
$=[2(x-1)]^2-(y+2)^2$
$=[2(x-1)+(y+2)][2(x-1)-(y+2)]$
$=[2x-2+y+2][2x-2-y-2]$
$=(2x+y)(2x-y-4)$

Extra Help

1. $10x^2+11x+3$

Step 1: First coefficient times constant = $10 \times 3 = 30$.

Step 2: Decompose coefficient of middle term, $11x$.
You get $11x = 5x + 6x$ because $5 + 6 = 11$ and $5 \times 6 = 30$.

Step 3: Express middle term as $5x + 6x$ and remove common factors. You get $10x^2+5x+6x+3$

$$=5x(2x+1)+3(2x+1)$$

$$=(2x+1)(5x+3).$$

2. $10x^2-11x+3$

Step 1: First coefficient times constant $= 10 \times 3 = 30$.

Step 2: Decompose coefficient of middle term, $-11x$.
You get $-11x = -5x - 6x$ because $-5 - 6 = -11$ and $(-5)(-6) = 30$.

Step 3: Express middle term as $-5x - 6x$ and remove common factors. You get $10x^2-5x-6x+3$

$$=5x(2x-1)-3(2x-1)$$

$$=(2x-1)(5x-3).$$

3. $6x^2-7x-5$

Step 1: First coefficient times constant $= 6 \times -5 = -30$.

Step 2: Decompose coefficient of middle term, $-7x$.
You get $-7x = -10x + 3x$ because $-10 + 3 = -7$ and $-10 \times 3 = -30$.

Step 3: Express middle term as $-10x + 3x$ and remove common factors. You get $6x^2-10x+3x-5$

$$=2x(3x-5)+1(3x-5)$$

$$=(3x-5)(2x+1).$$

4. $6x^2+7x-5$

Step 1: First coefficient times constant $= 6 \times -5 = -30$.

Step 2: Decompose coefficient of middle term, $+7x$.
You get $7x = 10x - 3x$ because $10 - 3 = 7$ and $10 \times -3 = -30$.

Step 3: Express middle term as $10x - 3x$ and remove common factors. You get $6x^2+10x-3x-5$

$$=2x(3x+5)-1(3x+5)$$

$$=(3x+5)(2x-1).$$

Extensions

1. $30x^2y-80xy+40y$

$$=10y\left(3x^2-8x+4\right)$$

$$=10y(x-2)(3x-2)$$

2 $1-x^2+4x-4$

$$=1-\left(x^2-4x+4\right)$$

$$=1^2-(x-2)^2$$

$$=[1+(x-2)][1-(x-2)]$$

$$=(1+x-2)(1-x+2)$$

$$=(x-1)(3-x)$$

3. $x^2-14x+49=(x-7)^2$
Dimensions of the square are $(x-7) \times (x-7)$.
Perimeter of the square is $4(x-7)$.

Exploring Topic 4

Activity 1

Solve and verify simple quadratic equations by reducing to $x^2 = c, c > 0$.

1. $x^2 = 16$

$x = \pm\sqrt{16}$

$x = \pm 4$

Verify $x = 4$.

LS	RS
x^2	16
4^2	
16	

LS = RS

Verify $x = -4$.

LS	RS
x^2	16
$(-4)^2$	
16	

LS = RS

2. $x^2 = 64$

$x = \pm\sqrt{64}$

$x = \pm 8$

Verify $x = 8$.

LS	RS
x^2	64
8^2	
64	

LS = RS

Verify $x = -8$.

LS	RS
x^2	64
$(-8)^2$	
64	

LS = RS

3. $3x^2 = 75$

$$\frac{3x^2}{3} = \frac{75}{3}$$

$x^2 = 25$

$x = \pm\sqrt{25}$

$x = \pm 5$

Verify $x = 5$.

LS	RS
$3x^2$	75
$3(5)^2$	
3(25)	
75	

LS = RS

Verify $x = -5$.

LS	RS
$3x^2$	75
$3(-5)^2$	
3(25)	
75	

LS = RS

4. $7x^2 = 343$

$$\frac{7x^2}{7} = \frac{343}{7}$$

$x^2 = 49$

$x = \pm\sqrt{49}$

$x = \pm 7$

Verify $x = 7$.

LS	RS
$7x^2$	343
$7(7)^2$	
7(49)	
343	

LS = RS

Verify $x = -7$.

LS	RS
$7x^2$	343
$7(-7)^2$	
7(49)	
343	

LS = RS

5. $x^2 = 121$

$x = \sqrt{121}$ (positive only)

$= 11\text{cm}$

The dimensions of the square are 11 cm × 11 cm.

6. $x^2 = 21$

$x = \pm\sqrt{21}$

or

$x \doteq \pm 4.583$

Verify $x = \sqrt{21}$.

LS	RS
x^2	21
$\left(\sqrt{21}\right)^2$	
21	

LS = RS

Verify $x = -\sqrt{21}$.

LS	RS
x^2	21
$\left(-\sqrt{21}\right)^2$	
21	

LS = RS

7. $(x+4)^2 = 36$

$x + 4 = \sqrt{36}$

$x + 4 = 6$

$x = 2$

Side x is 2 cm in length.

8. $x^2 + 41 = 210$

$x^2 = 210 - 41$

$x^2 = 169$

$x = \sqrt{169}$

$x = 13$

The dimensions of the square are 13 cm by 13 cm.

9. $x^2 = 123$

$x = \pm\sqrt{123}$

$x \doteq \pm 11.09$

Verify $x = 11.09$

LS	RS
x^2	123
$(11.09)^2$	
123	

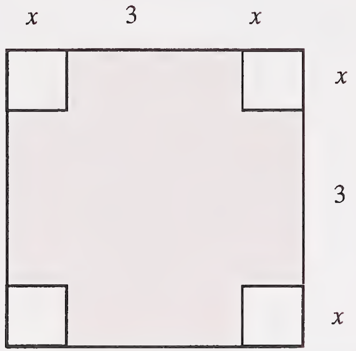
LS = RS

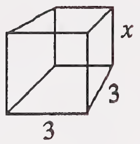
Verify $x = -11.09$.

LS	RS
x^2	123
$(-11.09)^2$	
123	

LS = RS

10.





$$(x+3+x)^2 = 225$$

$$(2x+3)^2 = 225$$

$$2x+3 = \pm\sqrt{225}$$

$$2x+3 = 15 \qquad \text{(positive solution only)}$$

$$2x = 12$$

$$x = 6$$

The height of the box is 6 cm.

Activity 2

Solve and verify simple quadratic equations by factoring.

1. a. $x^2 - 3x - 28 = 0$

$(x-7)(x+4) = 0$

$x-7=0 \qquad x+4=0$

$x=7 \qquad x=-4$

Verify $x = 7$.

LS	RS
$x^2-3x-28$	0
$(7)^2-3(7)-28$	
$49-21-28$	
0	

LS = RS

Verify $x = -4$.

LS	RS
$x^2-3x-28$	0
$(-4)^2-3(-4)-28$	
$16+12-28$	
0	

LS = RS

b. $x^2 - 9x - 36 = 0$

$(x-12)(x+3) = 0$

$x-12=0 \qquad x+3=0$

$x=12 \qquad x=-3$

Verify $x = 12$.

LS	RS
$x^2-9x-36$	0
$(12)^2-9(12)-36$	
$144-108-36$	
0	

LS = RS

Verify $x = -3$.

LS	RS
$x^2-9x-36$	0
$(-3)^2-9(-3)-36$	
$9+27-36$	
0	

LS = RS

2. a. $2x^2+9x-35=0$

$(x+7)(2x-5)=0$

$x+7=0 \qquad 2x-5=0$

$x=-7 \qquad 2x=5$

$x=\frac{5}{2}$

Verify $x=-7$.

LS	RS
$2x^2+9x-35$	0
$2(-7)^2+9(-7)-35$	
$98-63-35$	
0	

LS = RS

Verify $x=\frac{5}{2}$.

LS	RS
$2x^2+9x-35$	0
$2\left(\frac{5}{2}\right)^2+9\left(\frac{5}{2}\right)-35$	
$\frac{25}{2}+\frac{45}{2}-\frac{70}{2}$	
0	

LS = RS

2. b. $6x^2+11x-35=0$

$(2x+7)(3x-5)=0$

$2x+7=0 \qquad 3x-5=0$

$x=-\frac{7}{2} \qquad x=\frac{5}{3}$

Verify $x=-\frac{7}{2}$

LS	RS
$6x^2+11x-35$	0
$6\left(-\frac{7}{2}\right)^2+11\left(-\frac{7}{2}\right)-35$	
$6\left(\frac{49}{4}\right)-\frac{77}{2}-35$	
$\frac{147}{2}-\frac{77}{2}-\frac{70}{2}$	
0	

LS = RS

Verify $x=\frac{5}{3}$

LS	RS
$6x^2+11x-35$	0
$6\left(\frac{5}{3}\right)^2+11\left(\frac{5}{3}\right)-35$	
$6\left(\frac{25}{9}\right)+\frac{55}{3}-\frac{105}{3}$	
$\frac{50}{3}+\frac{55}{3}-\frac{105}{3}$	
0	

LS = RS

3. a. $10x^2+17x+3=0$

$(2x+3)(5x+1)=0$

$2x+3=0 \qquad 5x+1=0$

$2x=-3 \qquad 5x=-1$

$x=-\frac{3}{2} \qquad x=-\frac{1}{5}$

Verify $x=-\frac{3}{2}$.

LS	RS
$10x^2+17x+3$	0
$10\left(-\frac{3}{2}\right)^2+17\left(-\frac{3}{2}\right)+3$	
$10\left(\frac{9}{4}\right)-\frac{51}{2}+\frac{6}{2}$	
$\frac{45}{2}-\frac{51}{2}+\frac{6}{2}$	
0	

LS = RS

Verify $x=-\frac{1}{5}$.

LS	RS
$10x^2+17x+3$	0
$10\left(-\frac{1}{5}\right)^2+17\left(-\frac{1}{5}\right)+3$	
$10\left(\frac{1}{25}\right)-\frac{17}{5}+\frac{15}{5}$	
$\frac{2}{5}-\frac{17}{5}+\frac{15}{5}$	
0	

LS = RS

3. b. $20x^2+39x+7=0$

$(4x+7)(5x+1)=0$

$4x+7=0 \qquad 5x+1=0$

$4x=-7 \qquad 5x=-1$

$x=-\frac{7}{4} \qquad x=-\frac{1}{5}$

Verify $x = -\frac{7}{4}$.

LS	RS
$20x^2 + 39x + 7$	0
$20\left(-\frac{7}{4}\right)^2 + 39\left(-\frac{7}{4}\right) + 7$	
$20\left(\frac{49}{16}\right) - \frac{273}{4} + \frac{28}{4}$	
$\frac{245}{4} - \frac{273}{4} + \frac{28}{4}$	
0	

LS = RS

Verify $x = -\frac{1}{5}$.

LS	RS
$20x^2 + 39x + 7$	0
$20\left(-\frac{1}{5}\right)^2 + 39\left(-\frac{1}{5}\right) + 7$	
$20\left(\frac{1}{25}\right) - \frac{39}{5} + \frac{35}{5}$	
$\frac{4}{5} - \frac{39}{5} + \frac{35}{5}$	
0	

LS = RS

4. $x(x+2) = 15$

$$x^2 + 2x = 15$$
$$x^2 + 2x - 15 = 0$$
$$(x+5)(x-3) = 0$$

$x + 5 = 0$ $\quad$ $x - 3 = 0$

$x = -5$ $\quad$ $x = 3$

(not positive) $\quad$ $x + 2 = 5$

Discard.

The two numbers are 3 and 5.

5. $x^2 = (x-3)^2 + (x-6)^2$

$$x^2 = x^2 - 6x + 9 + x^2 - 12x + 36$$

$x^2 - 18x + 45 = 0$

$(x-15)(x-3) = 0$

$x - 15 = 0$

$x = 15$

$x - 3 = 0$

$x = 3$

The value $x = 3$ is inadmissible, since it results in a negative value for a distance, and a negative value for a distance is not defined.

The value of x is 15.

6. Let x = width.
$x + 5$ = length.

$$(x)(x+5)=24$$
$$x^2+5x=24$$
$$x^2+5x-24=0$$
$$(x+8)(x-3)=0$$

$x+8=0$ $\quad$ $x-3=0$
$x=-8$ $\quad$ $x=3$

Discard – 8 because width
cannot be negative.

$x+5=8$
The length of the rectangle is 8 cm.

7. Let x = one number.
$x - 7$ = other number.

$$x(x-7)=18$$
$$x^2-7x=18$$
$$x^2-7x-18=0$$
$$(x-9)(x+2)=0$$

$x-9=0$ $\quad$ $x+2=0$
$x=9$ $\quad$ $x=-2$ (not positive) Discard.

$x-7=9-7$
$=2$

The two numbers are 9 and 2.

Extra Help

1. a. $81-4y^2$
$=(9)^2-(2y)^2$
$=(9+2y)(9-2y)$

b. $9-4x^2$
$=(3)^2-(2x)^2$
$=(3+2x)(3-2x)$

2. a. $1-(x+3)^2$
$=[1+(x+3)][1-(x+3)]$
$=(1+x+3)(1-x-3)$
$=(4+x)(-2-x)$

b. $(3y+2)^2-1$
$=[(3y+2)+1][(3y+2)-1]$
$=(3y+3)(3y+1)$

3. a. $18x^2-50$
$=2(9x^2-25)$
$=2(3x+5)(3x-5)$

b. $72-2y^2$
$=2(36-y^2)$
$=2(6+y)(6-y)$

4. a. $(x+1)^2-(x-1)^2$
$=[(x+1)+(x-1)][(x+1)-(x-1)]$
$=(x+1+x-1)(x+1-x+1)$
$=(2x)(2)$
$=4x$

b. $(x-3)^2-(x+1)^2$
$=[(x-3)+(x+1)][(x-3)-(x+1)]$
$=(x-3+x+1)(x-3-x-1)$
$=(2x-2)(-4)$
$=-8(x-1)$

Extensions

1. a. Factor $49x^2-25y^2$.

$$\begin{aligned}49x^2-25y^2 &= (7x)^2-(5y)^2\\ &= (7x+5y)(7x-5y)\end{aligned}$$

b. Factor x^2-16y^2.

$$\begin{aligned}x^2-16y^2 &= x^2-(4y)^2\\ &= (x+4y)(x-4y)\end{aligned}$$

2. a. Factor $27mn^4-12m^3$.

$$\begin{aligned}27mn^4-12m^3 &= 3m\left(9n^4-4m^2\right)\\ &= 3m\left[\left(3n^2\right)^2-(2m)^2\right]\\ &= 3m\left(3n^2+2m\right)\left(3n^2-2m\right)\end{aligned}$$

b. Factor $8x^3y^2-2x$.

$$\begin{aligned}8x^3y^2-2x &= 2x\left(4x^2y^2-1\right)\\ &= 2x\left[(2xy)^2-1^2\right]\\ &= 2x(2xy+1)(2xy-1)\end{aligned}$$

3. a. Factor $15x^2y+40xy^2+20y^3$.

$$\begin{aligned}15x^2y+40xy^2+20y^3 &= 5y\left[3x^2+8xy+4y^2\right]\\ &= 5y(x+2y)(3x+2y)\end{aligned}$$

b. Factor $6x^2y+16xy^2+10y^3$.

$$\begin{aligned}6x^2y+16xy^2+10y^3 &= 2y\left[3x^2+8xy+5y^2\right]\\ &= 2y(x+y)(3x+5y)\end{aligned}$$

4. a. Factor $x^2-14xy+49y^2$.

$$\begin{aligned}x^2-14xy+49y^2 &= x^2-2(x)(7y)+(7y)^2\\ &= (x-7y)^2\end{aligned}$$

b. Factor $18x^2-60xy+50y^2$.

$$\begin{aligned}18x^2-60xy+50y^2 &= 2\left(9x^2-30xy+25y^2\right)\\ &= 2(3x-5y)^2\end{aligned}$$

5. a.

$$\frac{x+1}{2}+\frac{1}{x}=x+1$$

$$\overset{1}{\cancel{2}}x\left(\frac{x+1}{\underset{1}{\cancel{2}}}\right)+2\overset{1}{\cancel{x}}\left(\frac{1}{\underset{1}{\cancel{x}}}\right)=(2x)(x+1)$$

$$x(x+1)+2(1)=(2x)(x+1)$$

$$x^2+x+2=2x^2+2x$$

$$0=2x^2-x^2+2x-x-2$$

$$x^2+x-2=0$$

$$(x-1)(x+2)=0$$

$$x-1=0 \qquad x+2=0$$

$$x=1 \qquad x=-2$$

Verify $x = 1$.

LS	RS
$\frac{x+1}{2}+\frac{1}{x}$	$x+1$
$\frac{1+1}{2}+\frac{1}{1}$	$1+1$
$\frac{2}{2}+\frac{1}{1}$	2
$1+1$	
2	

LS = RS

Verify $x = -2$.

LS	RS
$\frac{x+1}{2}+\frac{1}{x}$	$x+1$
$\frac{-2+1}{2}+\frac{1}{-2}$	$-2+1$
$\frac{-1}{2}-\frac{1}{2}$	-1
-1	

LS = RS

5. b. $\frac{x-2}{3}+\frac{1}{2x}=x-\frac{1}{2}$ Multiply every term by $6x$.

$$\left(\overset{2}{\cancel{6}}x\right)\frac{x-2}{\underset{1}{\cancel{3}}}+\left(\overset{3}{\cancel{6x}}\right)\frac{1}{\underset{1}{\cancel{2x}}}=(6x)x-\left(\overset{3}{\cancel{6}}x\right)\frac{1}{\underset{1}{\cancel{2}}}$$

$$(2x)(x-2)+3=6x^2-3x$$

$$2x^2-4x+3=6x^2-3x$$

$$4x^2+x-3=0$$

$$(x+1)(4x-3)=0$$

$x+1=0$ $\quad$ $4x-3=0$

$x=-1$ $\quad$ $4x=3$

$x=\frac{3}{4}$

Verify $x = -1$.

LS	RS
$\frac{x-2}{3}+\frac{1}{2x}$	$x-\frac{1}{2}$
$\frac{-1-2}{3}+\frac{1}{2(-1)}$	$-1-\frac{1}{2}$
$\frac{-3}{3}+\frac{1}{-2}$	$-1\frac{1}{2}$
$-1+\frac{-1}{2}$	
$-1\frac{1}{2}$	

LS = RS

Verify $x = \frac{3}{4}$.

LS	RS
$\frac{x-2}{3}+\frac{1}{2x}$	$x-\frac{1}{2}$
$\frac{\frac{3}{4}-2}{3}+\frac{1}{2\left(\frac{3}{4}\right)}$	$\frac{3}{4}-\frac{1}{2}$
$\frac{\frac{-5}{4}}{3}+\frac{4}{6}$	$\frac{1}{4}$
$\frac{-5}{12}+\frac{8}{12}$	
$\frac{3}{12}$	
$\frac{1}{4}$	

LS = RS

6. a. Let x and $(x + 1)$ be the two consecutive numbers.

$$(x+1)^2+3x=37$$

$$x^2+2x+1+3x=37$$

$$x^2+5x-36=0$$

$$(x+9)(x-4)=0$$

$x+9=0$ $\quad$ $x-4=0$

$x=-9$ and $x+1=-8$ $\quad$ $x=4$ and $x+1=5$

The two numbers are – 8 and – 9 or 4 and 5.

b. Let x = smaller number.
$(x + 1)$ = larger number.

$$(x+1)^2-2x=2$$
$$x^2+2x+1-2x=2$$
$$x^2+1=2$$
$$x^2=1$$
$$x=\pm\sqrt{1}$$
$$x=\pm 1$$

If $x = 1$, $x + 1 = 2$.
If $x = -1$, $x + 1 = 0$.
The two numbers are 1 and 2 or –1 and 0.

7. a. The area of the swimming pool can be expressed in two ways.

1. $\text{Area} = 30 \text{ m} \times 40 \text{ m} \times \frac{1}{2}$
   $= 600 \text{ m}^2$

2. $\text{Area} = (40-2x)(30-2x)$

Since these two areas represent the same pool, they must be equal.

$$(40-2x)(30-2x)=600$$
$$1200-80x-60x+4x^2=600$$
$$4x^2-140x+600=0$$
$$4\left(x^2-35x+150\right)=0$$
$$4(x-30)(x-5)=0$$

$x-30=0$ $\quad$ $x-5=0$

$x=30$ Discard because $(30 – 2x)$ is negative. $\quad$ $x=5$

So $(40 – 2 \times 5) = 30$ and $(30 – 2 \times 5) = 20$.
The dimensions of the swimming pool are 30 m by 20 m.

b.
$$(x+5)(x+8)=70$$
$$x^2+5x+8x+40=70$$
$$x^2+13x-30=0$$
$$(x+15)(x-2)=0$$

$x+15=0$ $\quad$ $x-2=0$

$x=-15$ $\quad$ $x=2$

$x+5=-10$ $\quad$ $x+5=7$

$x+8=-7$ (Discard). $\quad$ $x+8=10$

The new dimensions of the rectangle are 7 cm × 10 cm.

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